

# Nathan M. Dunfield

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Area: Topology and geometry of 3-manifolds and related topics.

Personal: Born in Ann Arbor, Michigan in 1975. U.S. citizen.

# **Education:**

University of Chicago: Ph.D. in Mathematics, 1999. Advisors: Peter Shalen and Mel Rothenberg.

Oregon State University: B.S. in Mathematics, summa cum laude, 1994.

# **Employment:**

University of Illinois at Urbana-Champaign: Professor of Mathematics, 2018–present. University of Illinois at Urbana-Champaign: Associate Professor of Mathematics, 2007–2018. Caltech: Associate Professor of Mathematics, 2003–2007. Harvard: Benjamin Peirce Assistant Professor of Mathematics, 1999–2003.

# Awards and honors:

Simons Fellowship in Mathematics, Fall 2013 and Spring 2021.
Fellow of the American Mathematical Society (inducted 2013).
Alfred P. Sloan Fellow, 2004–2008.
Institute memberships (full semester): MSRI (Fall 2020, as research professor), IAS (Fall 2015), ICERM (Fall 2013).
Campus Distinguished Promotion Award, Univ. of Illinois, 2018.
Associate of the Center for Advanced Study, Univ. of Illinois, Fall 2017.
Wolfgang Haken Scholar, Dept. of Mathematics, Univ. of Illinois, 2021–2023.
LAS Dean's Award for Excellence in Undergraduate Teaching, Univ. of Illinois, 2014.
Distinguished Teaching Award, Dept. of Mathematics, Univ. of Illinois, 2012.
Faculty Teaching Award from the Associated Students of Caltech, 2006.
NSF Mathematical Sciences Postdoctoral Fellow, 2000–2003.
Alfred P. Sloan Dissertation Fellow, 1998–1999.
NSF Graduate Fellow, 1995–1998.

# **National Science Foundation Grants:**

DMS-2303572 "Interactions between geometry, topology, number theory, and dynamics", \$400,000 (sole PI).

DMS-1811156 "Facets of the Topology and Geometry of 3-Manifolds", \$365,000 (sole PI). Includes AGEP-GRS supplements awarded in 2021 and 2022.

DMS-1510204, "Facets of low-dimensional topology", \$390,000 (sole PI).

DMS-1105476, "Facets of the topology and geometry of 3-manifolds", \$185,000 (sole PI).

DMS-0707136, "Surfaces in finite covers of 3-manifolds and aspects of the mapping class groups", \$280,000 (sole PI).

DMS-0405491 "Groups and Group Actions in Low-Dimensional Topology", \$260,000 (co-PI with D. Calegari).

## **Publications:** Available on web page listed above, and at arXiv.org via the below clickable links.

- (with N. Brown and G. Perry) Colorings of the Plane, Parts I, II, and III. *Geombinatorics* **3** (1993), 24–31, **3** (1994), 64–74, **3** (1994), 110–114.
- Examples of non-trivial roots of unity at ideal points of hyperbolic 3-manifolds. *Topology* **38** (1999), 457–465. arXiv:math/9801064
- Cyclic surgery, degrees of maps of character curves, and volume rigidity for hyperbolic manifolds. *Invent. Math.* **136** (1999), 623–657. arXiv:math/9802022

A table of boundary slopes of Montesinos knots. *Topology*, **40** (2001), 309–315. arXiv:math/9901120

Alexander and Thurston norms of fibered 3-manifolds. *Pacific J. Math.* **200** (2001), 43–58. arXiv:math/9908050

(with Danny Calegari) Commensurability of 1-cusped hyperbolic 3-manifolds. *Trans. Amer. Math. Soc.* **354** (2002), 2921–2932. arXiv:math/0102024

(with Danny Calegari) Laminations and groups of homeomorphisms of the circle. *Invent. Math.* **152** (2003), 149–207. arXiv:math/0203192

(with William Thurston) The Virtual Haken Conjecture: Experiments and examples. *Geom. Topol.* **7** (2003), 399–441. arXiv:math/0209214

The Mahler measure of the *A*-polynomial of m129(0,3). 9 pages. Appendix to D. Boyd and F. Rodriguez Villegas, Mahler's measure and the dilogarithm (II). Preprint, 2003. arXiv:math/0308041

- (with Stavros Garoufalidis) Non-triviality of the *A*-polynomial for knots in S<sup>3</sup>. *Algebr. Geom. Topol.* **4** (2004), 1145–1153. arXiv:math/0405353
- (with Danny Calegari) An ascending HNN extension of a free group inside  $SL(2, \mathbb{C})$ . *Proc. Amer. Math. Soc.* **134** (2006), 3131–3136. arXiv:math/0412136
- (with Sergei Gukov and Jacob Rasmussen) The superpolynomial for knot homologies. *Experimental Math.* **15** (2006), 129–159. arXiv:math/0505662
- (with Frank Calegari) Automorphic forms and rational homology 3-spheres. *Geom. Topol.* **10** (2006), 295–329. arXiv:math/0508271
- (with William Thurston) Finite covers of random 3-manifolds.

Invent. Math. 166 (2006), 457-521. arXiv:math/0502567

- (with Dylan Thurston) A random tunnel number one 3-manifold does not fiber over the circle. *Geom. Topol.* **10** (2006), 2431–2499. arXiv:math/051029
- Volume change under drilling: theory vs. experiment. 5 pages. Appendix to Agol, Storm, and W. Thurston, Lower bounds on volumes of hyperbolic Haken 3-manifolds.
  J. Amer. Math. Soc. 20 (2007), 1053–1077. arXiv:math.DG/0506338
- (with S. Garoufalidis, A. Shumakovitch, and M. Thistlethwaite) Behavior of knot invariants under genus 2 mutation. *New York J. Math.* **16** (2010), 99–123. arXiv:math.GT/0607258
- (with Dinakar Ramakrishnan) Increasing the number of fibered faces of arithmetic hyperbolic 3manifolds. *Amer. J. Math* **132** (2010), 53–97. arXiv:0712.3243
- (with John W. Aaber) Closed surface bundles of least volume. Algebr. Geom. Topol. 10 (2010), 2315–2342. arXiv:1002.3423
- (with Helen Wong) Quantum invariants of random 3-manifolds. *Algebr. Geom. Topol.* **11** (2011), 2191–2205. arXiv:1009.1653
- (with Anil Hirani) The Least Spanning Area of a Knot and the Optimal Bounding Chain Problem. *Proceedings of the 27th annual ACM Symposium on Computational Geometry*, SoCG 2011, 135–144. arXiv:1012.3030
- (with Stavros Garoufalidis) Incompressibility criteria for spun-normal surfaces. *Trans. Amer. Math. Soc.* **64** (2012), 6109–6137. arXiv:1102.4588
- (with Stefan Friedl and Nicholas Jackson) Twisted Alexander polynomials of hyperbolic knots. *Experimental Math.* **21** (2012), 329–352. arXiv:1108.3045
- (with Jeffrey Brock) Injectivity radii of hyperbolic integer homology 3-spheres. *Geom. Topol.* **19** (2015), 497–523. arXiv:1304.0391
- (with Neil Hoffman and Joan Licata) Asymmetric hyperbolic *L*-spaces, Heegaard genus, and Dehn filling. *Math. Res. Letters* **22** (2015), 1679–1698. arXiv:1407.7827
- A diffuse group which is not left-orderable. Appendix to Kionke and Raimbault, On geometric aspects of diffuse groups. *Documenta Math.* **21** (2016), 873–915. arXiv:1411.6449
- (with Ian Agol) Certifying the Thurston norm via SL(2, ℂ)-twisted homology.
  What's next? The Mathematical Legacy of William P. Thurston,
  Ann. of Math. Studies 205 (2020), 1–20, Princeton Univ. Press. arXiv:1501.02136
- A knot without a nonorientable essential spanning surface. *Illinois J. Math.* **60** (2016), 179–184. arXiv:1509.06653
- (with Jeffrey Brock) Norms on the cohomology of hyperbolic 3-manifolds. *Invent. Math.* **210** (2017), 531–558. arXiv:1510.06292
- (with Marc Culler) Orderability and Dehn filling. *Geom. Topol.* **22** (2018), 1405–1457. arXiv:1602.03793
- A census of exceptional Dehn fillings. *Characters in low-dimensional topology*, Contemp. Math. **760** (2020), 143–155. arXiv:1812.11940
- Floer homology, group orderability, and taut foliations of hyperbolic 3-manifolds. *Geom. Topol.* **24** (2020), 2075–2125. arXiv:1904.04628

- (with Stavros Garoufalidis and J. Hyam Rubinstein) Counting essential surfaces in 3-manifolds. *Invent. Math.* **228** (2022), 717–775. arXiv:2007.10053
- S. Peddada, N. Dunfield, L. Zeidner, K. James, and J. Allison, Systematic Enumeration and Identification of Unique Spatial Topologies of 3D Systems Using Spatial Graph Representations. *Proceedings of the ASME 2021 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, Volume 3A: 47th Design Automation Conference (DAC), 13 pages. arXiv:2107.13724v1
- (with Cameron Rudd and Malik Obeidin) Computing a Link Diagram from its Exterior.
   Proceedings version: *Proceedings of the 38th International Symposium on Computational Geometry (SoCG 2022)*, pages 37:1–37:24.
   Journal version: *Discrete & Computational Geometry* (to appear), 34 pages. arXiv:2112.03251
- (with Jacob Rasmussen) A unified Casson-Lin invariant for the real forms of SL(2). Preprint 2022, 147 pages. arXiv:2209.03382
- (with Sherry Gong, Thomas Hockenhull, Marco Marengon, and Michael Willis) On the rank of knot homology theories and concordance. Preprint 2023, 27 pages. arXiv:2303.04233
- S. Peddada, N. Dunfield, L. Zeidner, Z. Givans, K. James, and J. Allison, Enumeration and Identification of Unique 3D Spatial Topologies of Interconnected Engineering Systems Using Spatial Graphs *Journal of Mechanical Design*, Oct 2023, 145(10): 101708 (16 pages). arXiv:2107.13724v2
- (with Robert Lipshitz and Dirk Schuetz) Local equivalence and refinements of Rasmussen's sinvariant. Preprint 2023, 43 pages. arXiv:2312.09114
- (with Giulio Tiozzo) Roots of Alexander polynomials of random positive 3-braids. Preprint, 49 pages. arXiv:2402.06771

# Talks:

# **Special Lectures:** The Dan E. Christie Mathematics Public Lecture at Bowdoin College, 2007. Invited address: AMS Sectional meeting in Raleigh, NC, 2009. Undergraduate Mathematics Symposium, UIC, October 2012. 33rd Annual Geometric Topology Workshop (principal speaker), Colorado Springs, June 2016. Rankin Lecture, University of Glasgow, December 2017. **Invited address:** AMS Sectional meeting in Madison, WI, September 2019. **Conferences:** Georgia Topology Conference, University of Georgia, August 1998. Workshop on Computation in Low-dimensional Topology, Oklahoma State, March 1999. Symposium on Computation in Group Theory and Geometry, University of Warwick, July 1999. Weekend Topology Conference, UC Berkeley, April 2000. 3-manifolds workshop, Barnard College, November 2000. Georgia Topology Conference, University of Georgia, May 2001. Workshop on groups and 3-manifolds, CRM, Montréal, June 2001. Topology in and around dimension three, Banff Research Station, September 2003. Trends in 3-manifolds, Université du Québec à Montréal, May 2004. Cornell Topology Festival, May 2004. Knots in Vancouver, University of British Columbia, July 2004. Low-dimensional topology, University of Virginia, December 2004. Geometry and Topology of 3-Manifolds, ICTP, Trieste, Italy, June 2005. Foundations of Computational Mathematics, Santander, Spain, July 2005. Pacific Northwest Geometry Seminar, Oregon State, November 2005. 3-manifold Topology in Honor of Peter Shalen's 60th Birthday, CRM, Montréal, June 2006. IAS/Park City Mathematics Institute, June 2006. Midwest Topology Conference, University of Illinois, October 2007. Interactions of Geometry and Topology, Banff Workshop, March 2009. Hamilton Geometry and Topology Workshop, Dublin, September 2009. Computational Differential Geometry, Topology, and Dynamics, Fields Institute, Nov. 2009. Bloomington Geometry Workshop, Indiana University, April 2010. Virtual properties of 3-manifolds, Université du Québec à Montréal, April 2010. Workshop on pseudo-Anosovs with small dilatation, UW Madison, April 2010. Topology and Geometry in Dimension Three, Oklahoma State, June 2010. Low-Dimensional Topology and Number Theory, Oberwolfach, August 2010. Geometry, analysis, and surfaces, Autrans, France, March 2011. ACM Symposium on Computational Geometry, INRIA, Paris, June 2011. Geometry & Topology Down Under, Melbourne, July 2011. 3-Manifolds, Artin Groups, and Cubical Geometry, CUNY Graduate Center, August 2011. Ordered Groups and Topology, Banff, February 2012. Redbud Topology Conference, Oklahoma State, March 2012.

Torsion in the homology of arithmetic groups, Banff, July 2012. GEAR Network Retreat, University of Illinois, August 2012. Midwest Topology Seminar, Michigan State, October 2012. Group Theory, Number Theory, and Topology Day, Madison, January 2013. Mapping Class Groups and Categorification, Banff, April 2013. The Topology of 3-dimensional Manifolds, CRM, Montréal, May 2013. Cube Complexes and 3-manifolds, UIC, May 2013. Geometric Topology in New York, Columbia, August 2013. Topology, Geometry and Group Theory, Informed by Experiment, ICERM, October 2013. Computational & Algorithmic Topology, Sydney, March 2014. What's Next? The mathematical legacy of Bill Thurston, Cornell, June 2014. Combinatorial Link Homology Theories, Braids, and Contact Geometry, ICERM, August 2014. Low-Dimensional Topology and Number Theory, Oberwolfach, August 2014. Random Spaces, AMS meeting special session, Eau Claire, September 2014. Texas Geometry and Topology Conference, Austin, November 2014. Tech Topology Conference, Georgia Tech, December 2014. Computability, Analysis, and Geometry, Banff, March 2015. Representations of 3-manifold groups, geometric structures and exact computations, Institut de Mathématiques de Jussieu, June 2015. Symposium on Geometry and Topology of Manifolds, PIMS, Vancouver, July 2015. Workshop on 3-manifolds, Warwick, July 2015. New developments in TQFT, QGM, Aarhus University, July 2015. Computational Geometric and Algebraic Topology, Oberwolfach, October 2015. Geometry of groups, surfaces and 3-manifolds, AMS special session, Rutgers, November 2015. Workshop on Flows, Foliations and Contact Structures, IAS, December 2015. Interactions of Gauge Theory with Contact and Symplectic Topology in Dimensions 3 and 4, Banff, March 2016. Graduate Student Topology and Geometry Conference, Indiana University, April 2016. Sage Days 74: Differential geometry and topology, Observatoire de Paris, May 2016. Geometric Group Theory and Low-Dimensional Topology: Recent Connections and Advances, ICTP, Trieste, Italy, June 2016. 33rd Annual Geometric Topology Workshop (principal speaker), Colorado Springs, June 2016. 3-manifold workshop, Newton Institute, Cambridge, January 2017. Shanks Conference on Low-dimensional Topology and Geometry, Vanderbilt, May 2017. Introduction to geometry, dynamics, and moduli in low dimensions, Warwick, September 2017. Geometric topology in low dimensions, Warwick, September 2017. Bloomington Geometry Workshop, Indiana University, April 2018. Characters in low-dimensional topology, CRM, Montréal, June 2018. Hyperbolic 3-manifolds and related topics, POSTECH, South Korea, June 2018. Growth in Topology and Number Theory: Volumes, Entropy, and  $L^2$ -torsion, Hausdorff Inst., Bonn, July 2018.

Modularity and 3-manifolds, ICERM, March 2019.

- Three-dimensional Floer Theory, Contact Geometry, and Foliations, AMS meeting special session, Honolulu, March 2019.
- Georgia Topology Conference, University of Georgia, May 2019.
- Ordered Groups and Rigidity in Dynamics and Topology, CMO, Oaxaca, June 2019.
- IAS/Park City Mathematics Institute, July 2019.
- Perspectives on Dehn Surgery, ICERM, July 2019.
- Illustrating Geometry and Topology, ICERM, September 2019.
- Low-dimensional Topology, Oberwolfach, February 2020.
- Redbud Topology Festival, University of Arkansas, March 2020.
- Computational Aspects of Discrete Subgroups of Lie Groups, ICERM (online), June 2021.
- Knots, Surfaces and 3-manifolds, CMO, Oaxaca (online), June 2021.
- Gauge Theory Workshop, Miami, April 2022.
- Arithmetic Groups and 3-Manifolds, Bonn, Germany, May 2022.
- Topology 22, Banaras Hindu University (online), December 2022.
- Low-dimensional Topology, Oberwolfach, January 2023.
- Computational Problems in Low-dimensional Topology III, Rutgers-Newark, April 2023.
- Midwest Topology Seminar, Purdue, April 2023.
- Orderable groups, CRM, Montréal, May 2023.
- Geometric Topology in New York (GTiNY), June 2023.
- 3-manifold groups, CRM, Montréal, June 2023.
- XXVth ISM Graduate Student Conference, Université de Sherbrooke, June 2023.
- Tech Topology Summer School, Georgia Tech, July 2023.
- Foliations and flows, AMS meeting special session, Omaha, October 2023.
- Michigan Intercity Topology Symposium, Ann Arbor, November 2023.
- Redbud Topology Festival, Oklahoma State, April 2024.
- Cornell Topology Festival, May 2024.
- Flows and Foliations in 3-manifolds, St. Louis, May 2024.

## Seminar Talks:

- 1999: Bay Area Topology Seminar (UC Davis), SUNY Stony Brook, SUNY Buffalo, Brown.
- 2000: Univ. of Michigan, Université du Québec à Montréal, SUNY Albany.
- 2001: Boston College, SUNY Buffalo.
- 2002: Univ. of Texas, Yale, Bay Area Topology Seminar (UC Davis), Caltech, Columbia.
- 2003: Stanford, Cornell, UC Santa Barbara, Univ. of British Columbia, Maryland, Univ. of Utah.
- 2004: UC Berkeley.
- 2005: Princeton, Columbia, Univ. of Southern California, UC Davis.
- 2006: UC Berkeley, UW Madison.
- 2007: Michigan State, UC Santa Barbara.
- 2008: Indiana University, UIC.
- 2009: Northwestern.

- 2011: UC Santa Barbara, Temple, Columbia, Brown, Harvard, Northwestern.
- 2012: UCLA, Stanford, Berkeley, LA Topology Seminar (at USC), Harvard, Chicago.
- 2013: UC Santa Barbara, UIC, Ohio State, Brown.
- 2014: Melbourne.
- 2015: Chicago, Stanford, UC Davis, Yale, CUNY, Princeton.
- 2016: UIC, Duke.
- 2017: Cambridge, MIT, Brandeis, Boston College, LA Topology Seminar, Glasgow, Oxford.
- 2019: Univ. of Toronto.

2020: Cambridge, Geometry and Topology Online (GaTO), Virtual Seminar on Geometry and Topology (visgat), Univ. of Texas (online), MSRI (online).

- 2021: UC Davis (online).
- 2022: Yau Center at Tsinghua University (online).

2023: Berkeley, UNV Reno, Stanford, Washington University, Univ. of Oregon.

#### **Departmental Colloquia:**

- 2000: SUNY Albany.
- 2002: Caltech, Georgia Tech.
- 2003: UIC, Univ. of Utah, UC Davis, Univ. of British Columbia, Univ. of Toronto.
- 2005: Columbia, UIUC, Oregon State.
- 2006: UW Madison, UC San Diego, Northeastern.
- 2007: UCLA, Michigan State, UIUC, UC Santa Cruz, Univ. of Oregon, Brown, Bowdoin.
- 2008: Indiana University.
- 2011: UIC, Temple, Tufts.
- 2013: Dartmouth.
- 2014: Melbourne.
- 2015: Columbia, IAS.
- 2016: Univ. of Oregon, UC Irvine.
- 2017: Northwestern, Penn State, Univ. of Pittsburgh, Tufts, Washington University, UCLA.
- 2018: Eastern Illinois Univ.
- 2019: Rice.
- 2021: Southern University of Science and Technology (online).
- 2023: Washington University.

## Software:

(with Marc Culler, Matthias Goerner, and Jeffery Weeks) *SnapPy*, a program for studying the topology and geometry of 3-manifolds. 2009–present. http://snappy.computop.org

## **Teaching:**

I have a broad range of teaching experience, having taught more than 20 distinct courses ranging from a vector calculus class with 270 students to an advanced graduate topics class with only 5 students. My courses have been consistently well-received; for instance, I won a Faculty Teaching

Award from the Associated Students of Caltech in 2006, a LAS Dean's Award for Excellence in Undergraduate Teaching in 2014, and was on the *List of Teachers Rated as Excellent* at Illinois 21 times, including six times for a 200+ person calculus class.

**Calculus:** At Illinois, I have taught 11 large vector calculus classes with between 170 and 270 students, with a total enrollment of 2,700. For this course, I developed active-learning worksheets for the discussion sections that are closely coupled to the lecture content; these worksheets have been adopted by many other instructors here, and so have been used by more than 20,000 students over the past ten years. At Caltech, I twice taught a 120–student vector calculus class aimed at science and engineering majors.

Advanced undergraduate classes: I have taught classes aimed at students concentrating in math and related fields in a number of areas: elementary number theory, linear algebra, abstract algebra, probability, topology, and differential geometry.

**Graduate classes:** I have taught first-year graduate classes both in my core area of topology and also in algebraic number theory. I have twice given a one-term introduction to Riemannian geometry. I have also given five graduate topics classes.

Mentoring: I have graduated nine PhD students:

Vaibhav Gadre (2010) was a BP at Harvard, and is now a reader at Glasgow.

Jonah Sinick (2011) is a data scientist who is CEO at Equator Therapeutics.

BoGwang Jeon (2013) was a Rita Assistant Professor at Columbia and is now a tenure-track assistant professor at POSTECH in Pohang, South Korea.

Brian Benson (2014) held postdoctoral positions at Kansas State and UC Riverside and is now a research scientist at Metron.

Xinghua Gao (2019) was a postdoc at the Korean Institute for Advanced Study and is now a tenture-track assistant professor at Hunan University.

Malik Obeidin (2019) is a software engineer at Google.

Xiaolong Hans Han (2021) is a postdoc at the Yau Math Center at Tsinghua University.

Cameron Rudd (2022) is a postdoc at the Max Planck Institute in Bonn.

Chaeryn Lee (2023).

I am currently the PhD advisor for two graduate students. I have mentored numerous undergraduates in research projects and honors theses.

## Service:

#### **Selected Departmental Committees:**

Altgeld/Illini renovation committee (2018–2020, 2021–2023; chair 2019–2020).

Promotion and tenure committee (2021–2023, chair for 2022–2023).

Executive committee (2009–2011, 2014–2016, and 2023–2025). This elected committee is responsible for

all budgetary decisions, faculty hiring, and the yearly faculty evaluation for raises.

Undergraduate affairs committee (2011–2013, chair for 2012–2013).

Chair of mathematics graduate admissions (Caltech, 2005–2007).

### **Off-campus Committees:**

AMS Council, Member at Large, 2017–2021.

AMS Committee on Publications, 2017–2021.

AMS Web Advisory Group, 2017–2019. AMS Centennial Fellowship Selection Committee 2018–2020. AMS Central Section Program Committee, 2013–15; chair for 2014–15. IMA SageMath Scientific Board, 2016–2018.

#### **Organizer for:**

Perspectives on Dehn Surgery, ICERM, graduate summer school, July 2019.

Computation in geometric topology, Warwick, December 2017.

Workshop on Geometric Structures on 3-Manifolds, IAS, October 2015.

Conference in honor of Daryl Cooper's 60th birthday, Berkeley, May 2015.

Geometry, Groups, and Dynamics Day, Rose-Hulman, 2014 and 2016; Purdue, 2018.

Lead organizer for the Fall 2013 semester program Geometry, Topology, and Dynamics

at the Institute for Computational and Experimental Research in Mathematics at Brown. Special session at the Raleigh AMS meeting in April 2009.

Conference in Honor of Peter Shalen's 60th Birthday, CRM, Montréal. June 2006.

N + 2nd Southern California Topology Conference. April 2005.

#### **Editorial boards**:

New York Journal of Mathematics, 2013–2020.