Ma 109b, HW 2, Final version: Due Wednesday, Jan 18

Lecture Notes: Since I have deviated so much from the text in the past few lectures, I have a put a copy of my lecture notes on the course webpage:

`http://www.its.caltech.edu/~dunfield/classes/2006/109b/`

TA: The TA for this term is again Dongping Zhuang. His office hour will be on Mondays from 4–5pm in 385 Sloan.

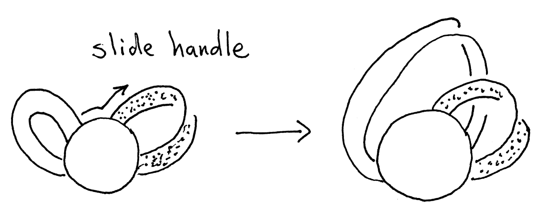
HW Note: You may assume that connected sum is well-defined when working these problems.

1. A handle decomposition of a compact surface S is called *simple* if there is only one 0-handle and one 2-handle. Given simple handle decompositions for surfaces S_1 and S_2, what is a simple handle decomposition for the connected sum $S_1 \# S_2$? Prove your answer.

2. Complete the proof of the Surface Classification Theorem by proving the following.

   Claim: Let S be a surface with a simple handle decomposition with n 1-handles. If at least one of the 1-handles is twisted, then S is homeomorphic to the connected sum of n copies of the projective plane.

   Caution: When you slide an untwisted handle over a twisted handle, the resulting handle is now twisted. Similarly, if you slide a twisted handle over a twisted handle you get an untwisted handle.



3. Let $\mathscr{H}$ be a handle decomposition of a compact connected surface S. Then define

$$\chi(\mathscr{H}) = v - e + f$$

   where v is the number of 0-handles, e the number of 1-handles, and f the number of 2-handles.

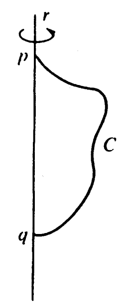
   (a) Prove that $\chi(\mathscr{H})$ does not depend on the choice handle decomposition, and thus it makes sense to talk about $\chi(S)$, the *Euler characteristic* of S.

   (b) Calculate $\chi(S)$ for each surface in the classification.

   Note: Taking the handle decomposition associated to a triangulation gives the version of this theorem that I mentioned on the first day of class, or equivalently, Theorem 3.5.2 in the text. You may not assume Theorem 3.5.2, or its main ingredient Corollary 3.7.3, in doing this problem.

4. From the text: 5.3.6

5. Let C be a regular plane curve which lies in one side of a straight line r of the plane and meets at the two points p, q as shown below. (For the definition of a *regular curve*, see the text at page 174). What conditions should C satisfy to ensure that the surface of revolution of C about r is a smooth surface? Prove your answer.



6. Let $S \subset \mathbb{R}^3$ be a smooth surface. Given $p \in S$, show that one can permute the coordinates so that there is a coordinate patch $f\colon U \to S$ which has the form $f(x, y) = (x, y, h(x, y))$ and with $p \in f(U)$.

   Note 1: Please use my definition of a coordinate patch, not the one in the text. That is, include the requirement that f is homeomorphism onto $f(U)$, and $f(U)$ is open in S.

   Note 2: Such a coordinate patch is called a Monge patch; see Section 5.3(1).