Ma 151b, HW #5, final version. Due Friday, February 10.

Important Note: The midterm will replace HW #6. It will be due on Friday, February 17th and will not be otherwise time restricted.

N1: Let M be a compact connected 3-manifold with a triangulation $\mathscr{T}$. Prove that each of for any k-simplex $\sigma \in \mathscr{T}$ the dual "cell" is really a cell. That is, prove that $(\overline{D}(\sigma), \dot{D}(\sigma)) \cong (B^{3-k}, \partial B^{3-k})$.

N2: Poincaré duality for 3-manifolds. Let M be a compact, connected orientable 3-manifold. The only interesting case of Poincaré duality for 3-manifolds is that $H^1(M, \mathbb{Z})$ is isomorphic to $H_2(M, \mathbb{Z})$. Fill in the following outline for part of a geometric proof (all (co)homology has coeffs in $\mathbb{Z}$); in a later HW you will complete the proof once we know cohomology is representable.

(a) Prove that any class x in $H_1(M)$ can be represented by an oriented embedded circle.

(b) Prove that any class y in $H_2(M)$ can be represented by an oriented embedded surface. That is, there is an embedded surface $S \subset M$ with $i_*([S]) = y$.

(c) There is a bilinear pairing $H_1(M) \otimes H_2(M) \to \mathbb{Z}$, namely the intersection product. If x is represented by an embedded circle and y is represented by an embedded surface with x and y intersecting transversely, this is just the number of times x crosses y, counted with signs. This gives a map from $H_2(M) \to H_1(M)^*$. Show that this map is injective. (You may assume the intersection product is well defined.)

In a later HW you will show that $H_2(M) \to H_1(M)^* \cong H^1(M)$ is surjective, completing the proof.

N3: A knot K in S^3 is the image of a smooth embedding $S^1 \hookrightarrow S^3$. Prove that there is an embedded orientable surface Σ in S^3 whose boundary is K. Such a surface is called a Seifert surface. Here are some examples:

Hint: Use Alexander and Lefschetz duality.

Hatcher: Section 4.1, #2 and 4.