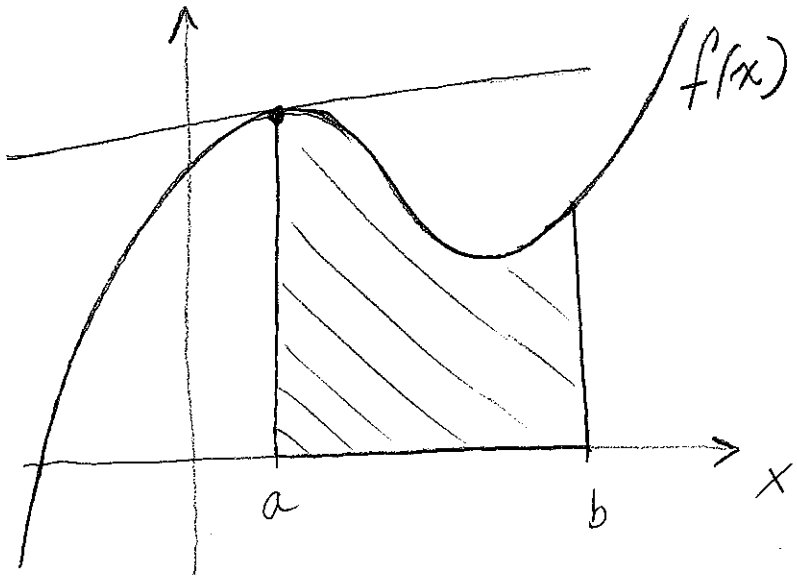
Math 241: Calculus III, Urbana, Fall 2010

Review in a picture

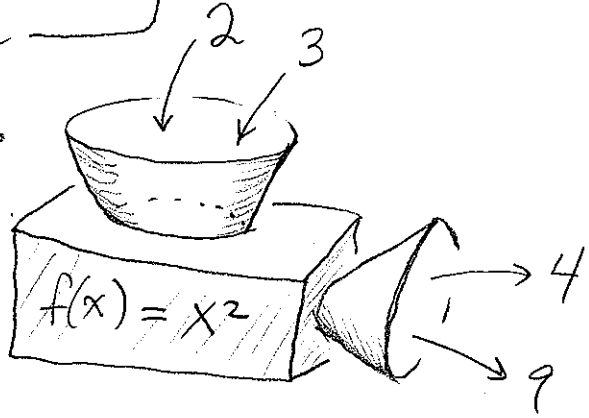
$f'(a)$ = slope of tangent line



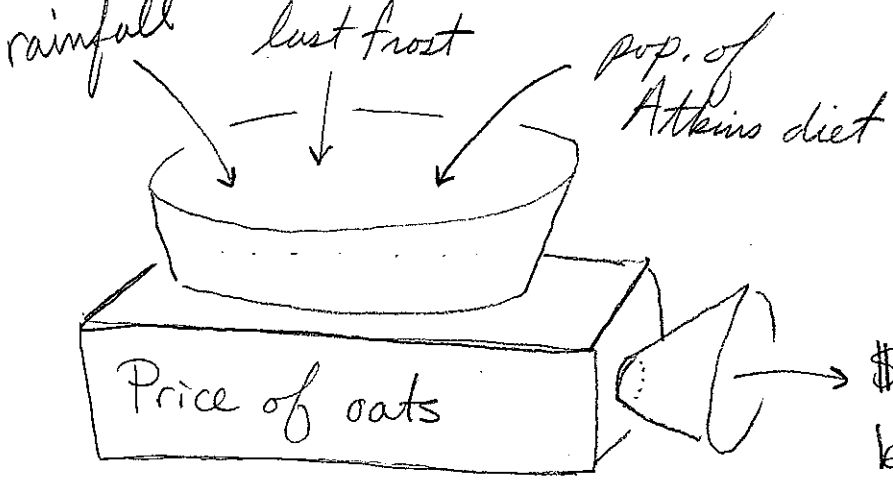
$$\text{Area} = \int_a^b f(x)\,dx = F(b) - F(a) \quad \text{with } F'(x) = f(x)$$

Fund. Thm. of Calc

Another way to think of $f(x)$:

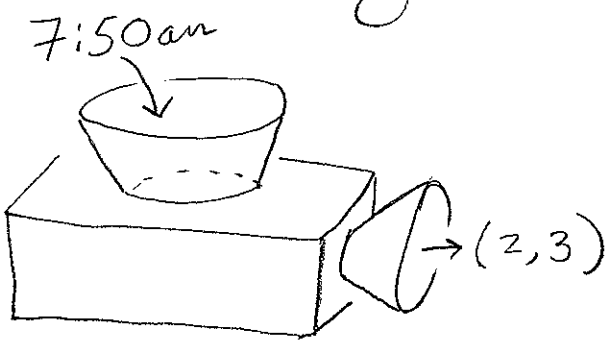


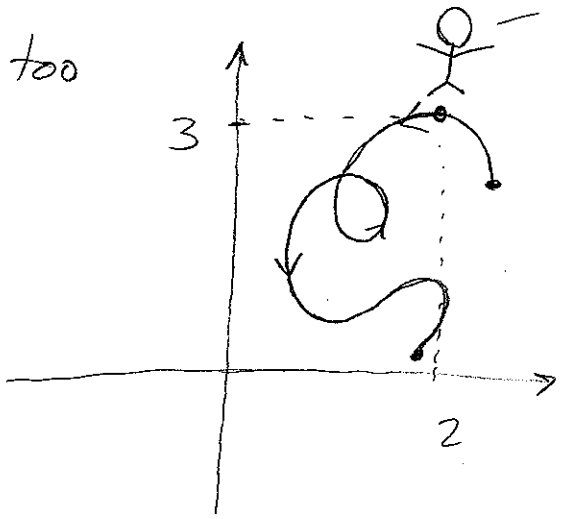
Real life is rarely so simple...



Goal: Understand such functions.

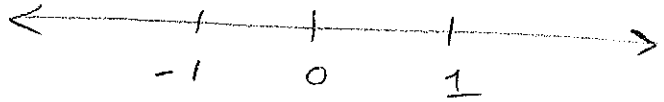
Can have many outputs, too



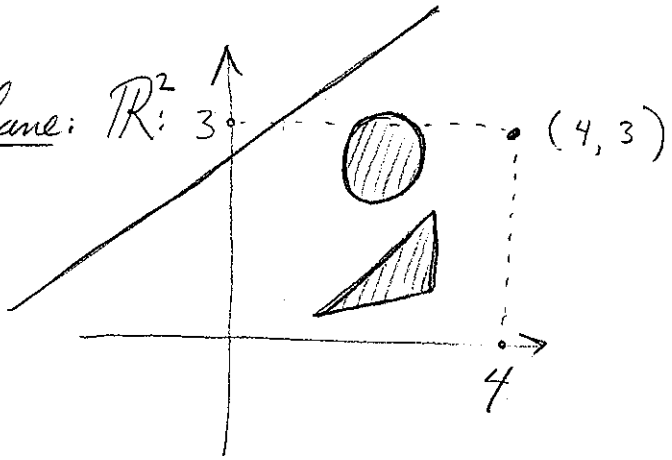


Basic concept: n-dimensional space (Section 12.1)

Line: $\mathbb{R}$ = real numbers

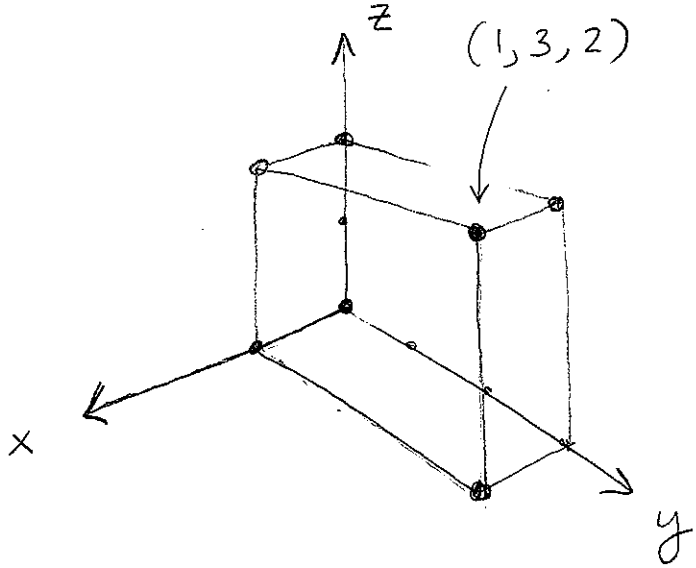


Plane: $\mathbb{R}^2$:



$\mathbb{R}^2$ = pairs (x,y) of real #s.

3-space: $\mathbb{R}^3$ – [the world we live in]



tripples (x,y,z) of #s.

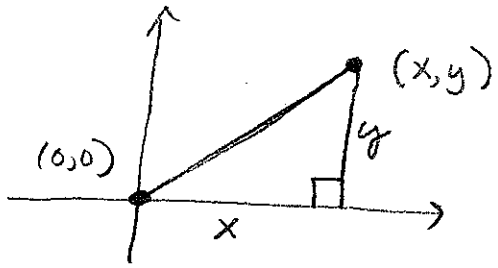
n-space: $\mathbb{R}^n$ = tuples $(x_1, x_2, x_3, \ldots, x_n)$

[In real world applications, n might be 10,000...]

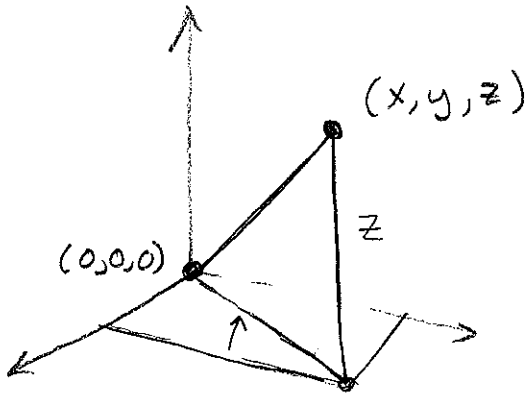
[You're familiar with $\mathbb{R}^2$ and will be with $\mathbb{R}^3$; the power of abstraction in math is that we can use our intuition from the simple to understand the complex.]

Distance:

In $\mathbb{R}^2$, distance between the shown points is $\sqrt{x^2+y^2}$



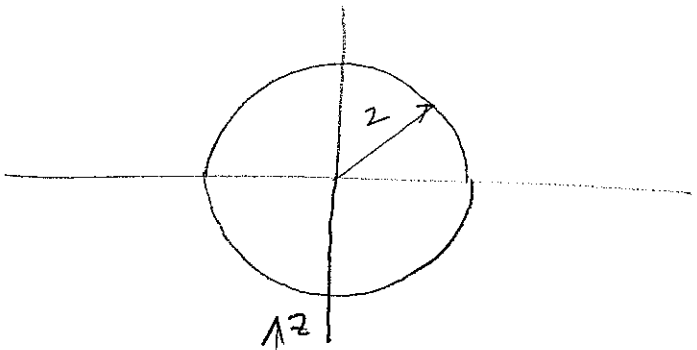
In $\mathbb{R}^3$, distance is $\sqrt{x^2+y^2+z^2}$



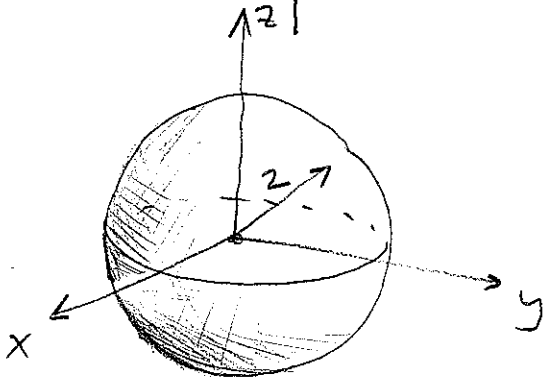
In $\mathbb{R}^n$ distance between $P(x_1, x_2, \ldots, x_n)$ and $Q(y_1, y_2, \ldots, y_n)$ is

$$|PQ| = \sqrt{(x_1-y_1)^2 + \cdots + (x_n-y_n)^2}$$

Ex: $x^2+y^2=4$ in $\mathbb{R}^2$



$x^2+y^2+z^2=4$ in $\mathbb{R}^3$



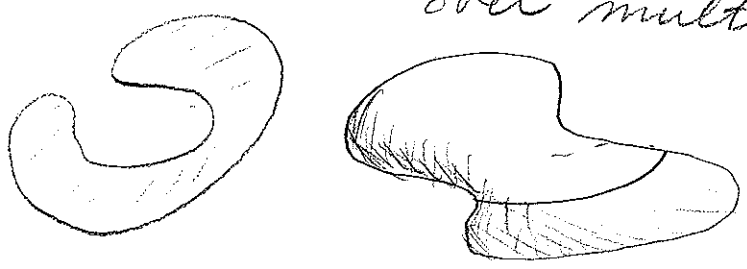
Outline of course:

- Vectors and geometry of $\mathbb{R}^n$ [planes, lines, dot and cross products.]
- Functions of several variables: Differentiation

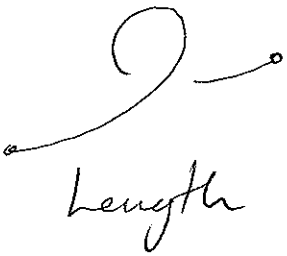
  Other ways of understanding f'

  a) Rate of change

  b) Taylor series $f(x)=f(0)+f'(0)x+\frac{f''(0)}{2}x^2+\cdots$
- Optimization (Min/Max)
- Integration: Areas, volumes, averages over multi-dimensional objects



- Curves and surfaces in $\mathbb{R}^3$.

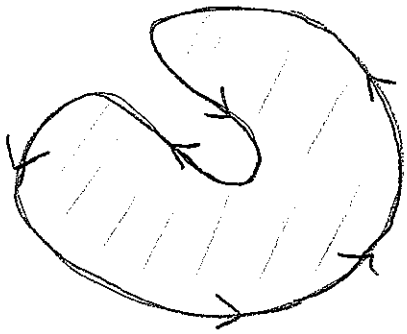


Length

Surface Area.

- Relationships between these, generalizing the Fund. Thm of Calculus.

E.g. Figuring out the area by integrating along the boundry. [Course highlight.]



Go over syllabus, etc.