Lecture 5: Functions of several variables (14.1)

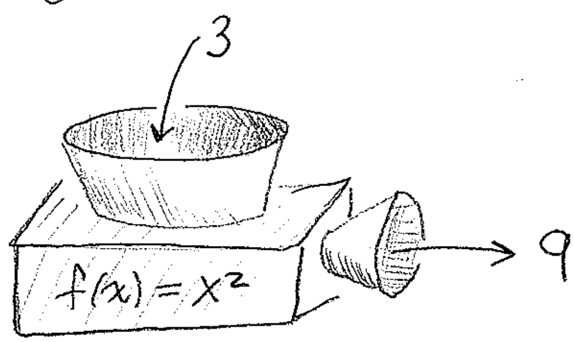
Office Hours: (Can go to any)

Nathan Tu 11-12:30 Th 4-5:30 in 378 Altgeld
Boonrod Tu 4-5:30 Th 11-12:30 in B1A Cobel
Ping MTW 2-3 in 224 Illini
Jeff TuTh 10-11, W 3-4 in B1 Cobel

28 hours of help each week.

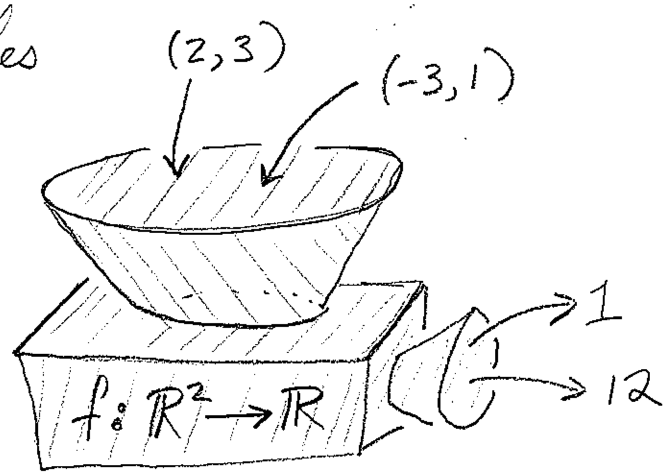
Evening Tutoring: MTuWTh from 5-9pm in 7 Illini

Function of one variable



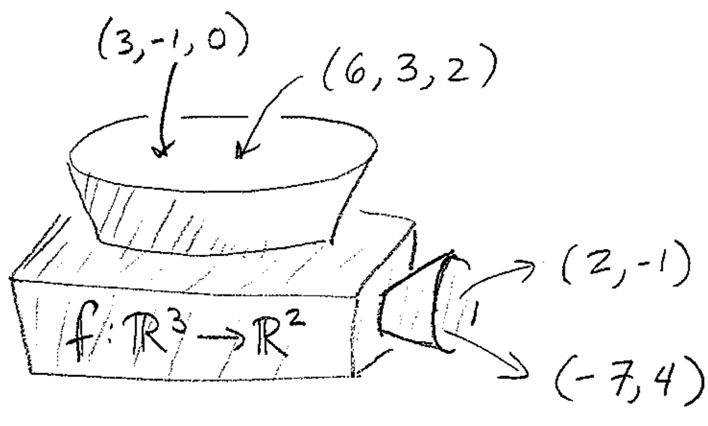
Function of 2 variables

Ex: $f(x,y) = x^2 - xy$



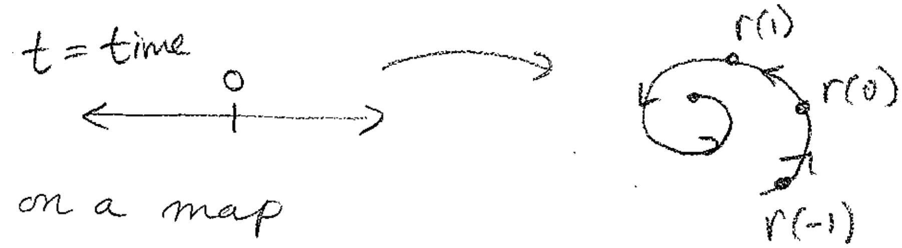
General function

$\mathbb{R}^n \longrightarrow \mathbb{R}^m$

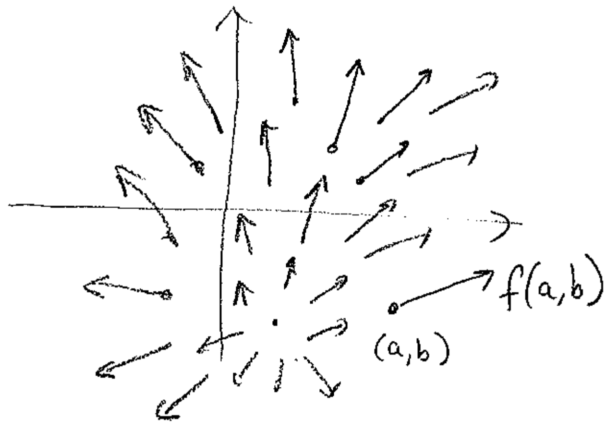


Ex: ① Temperature in this room $T: \mathbb{R}^3 \longrightarrow \mathbb{R}$

② Parameterized curve in the plane $r: \mathbb{R} \longrightarrow \mathbb{R}^2$



③ Wind speed / direction on a map

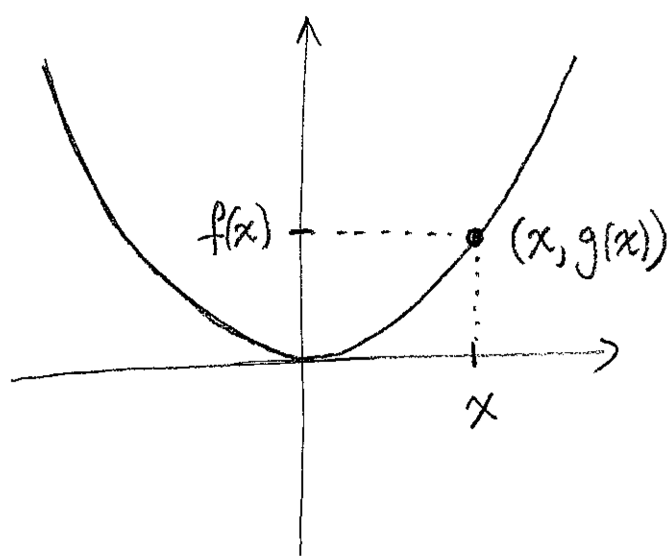


$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

[Eletric field, force due to gravity, etc.]

[Chapter 14 is ①, Chapt 13 is ②, and Chapter 16 includes 3.]

Today: Functions $\mathbb{R}^2 \longrightarrow \mathbb{R}$



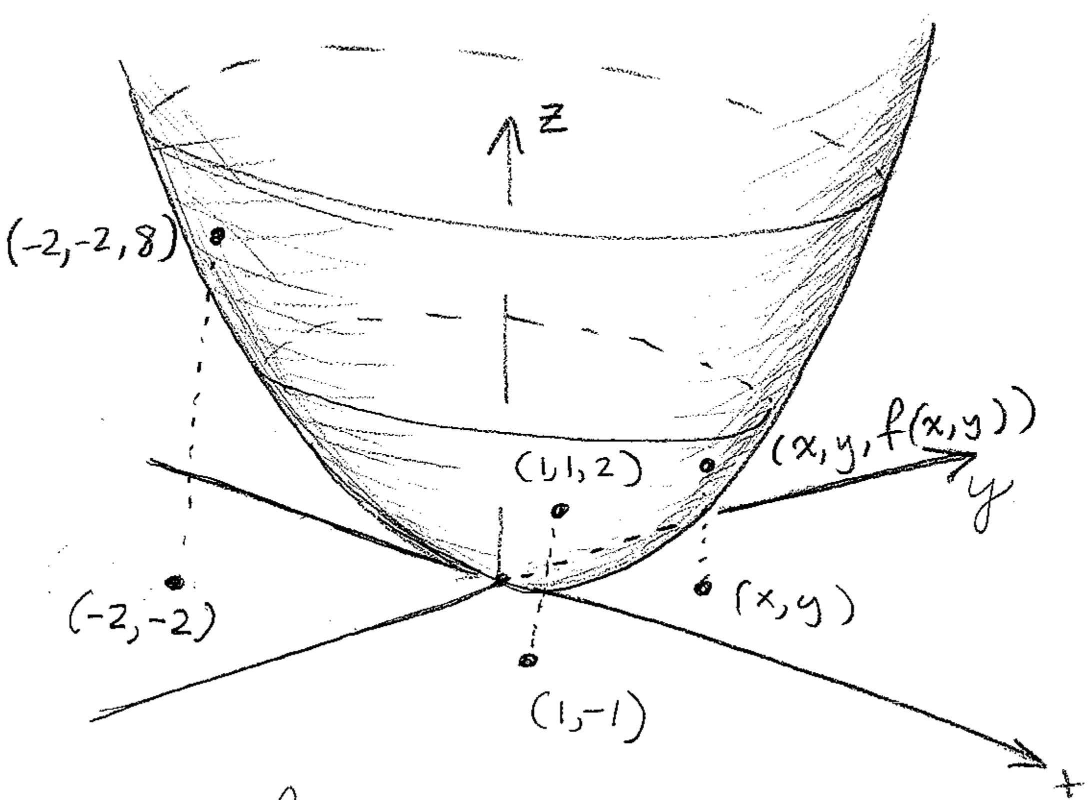
Graphs: One var $f: \mathbb{R} \longrightarrow \mathbb{R}$

$$f(x) = x^2$$

Consider $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$

given by $f(x,y) = x^2 + y^2$.

$$\text{Graph} = \{(x, y, f(x,y))\} \text{ in } \mathbb{R}^3$$

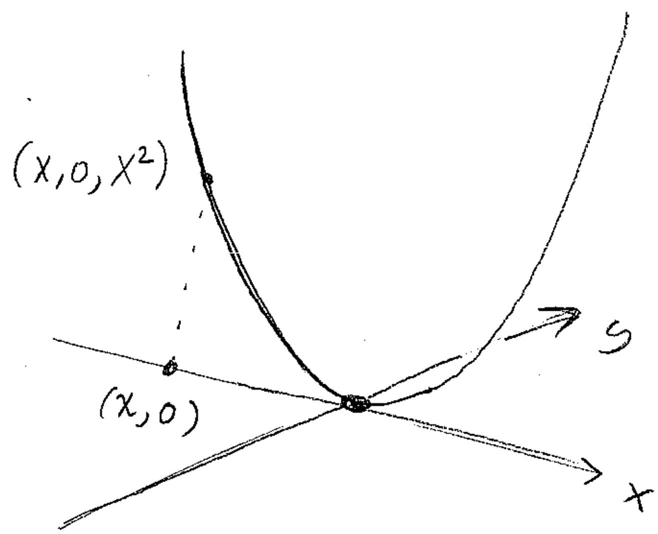


How to figure out:

Intersect with planes:

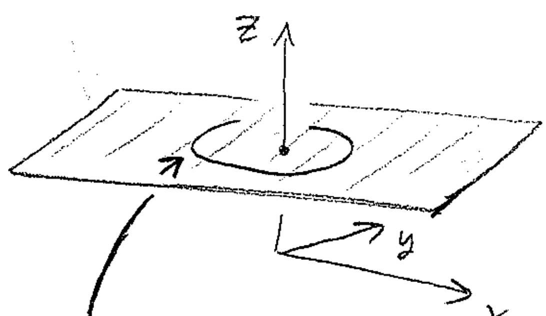
What is over the x-axis?

[or y-axis]



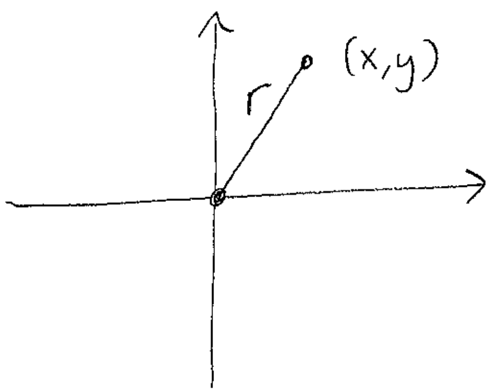
What is the intersection with $\{z = c\}$? Same finding all (x,y) with $f(x,y) = c$ that is

$$x^2+y^2=c$$



Circle of radius $\sqrt{c}$.

Symmetry: $x^2+y^2=r^2$



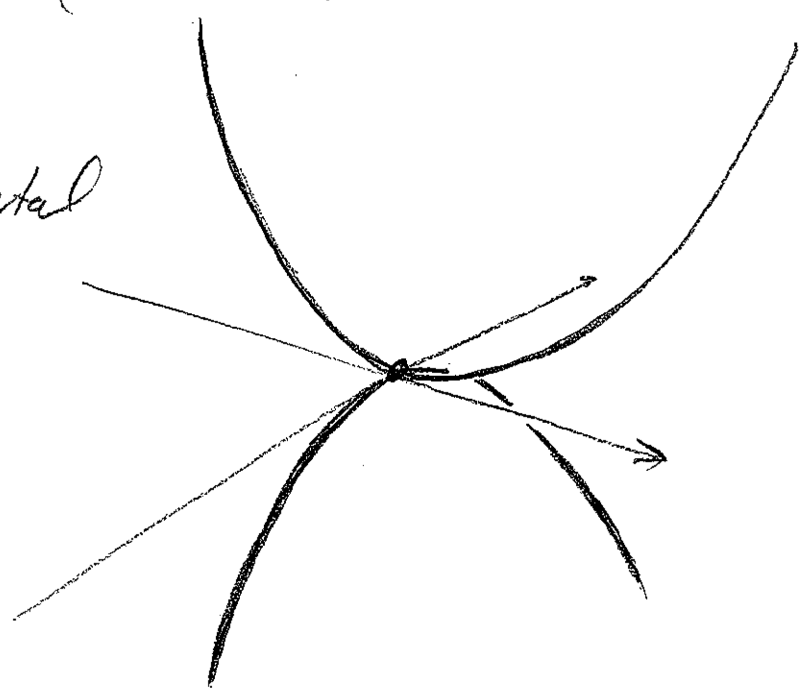
$\Longrightarrow$ Graph is invariant under rotation around the z-axis.

Computer: Can be useful.

Ex: $f:\mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = x^2 - y^2$

Over x-axis: $f(x,0) = x^2$ (parabola)

Over y-axis: $f(0,y) = -y^2$ (—— " ——)



Intersections with horizontal planes:

$z=0$: $x^2-y^2=0$

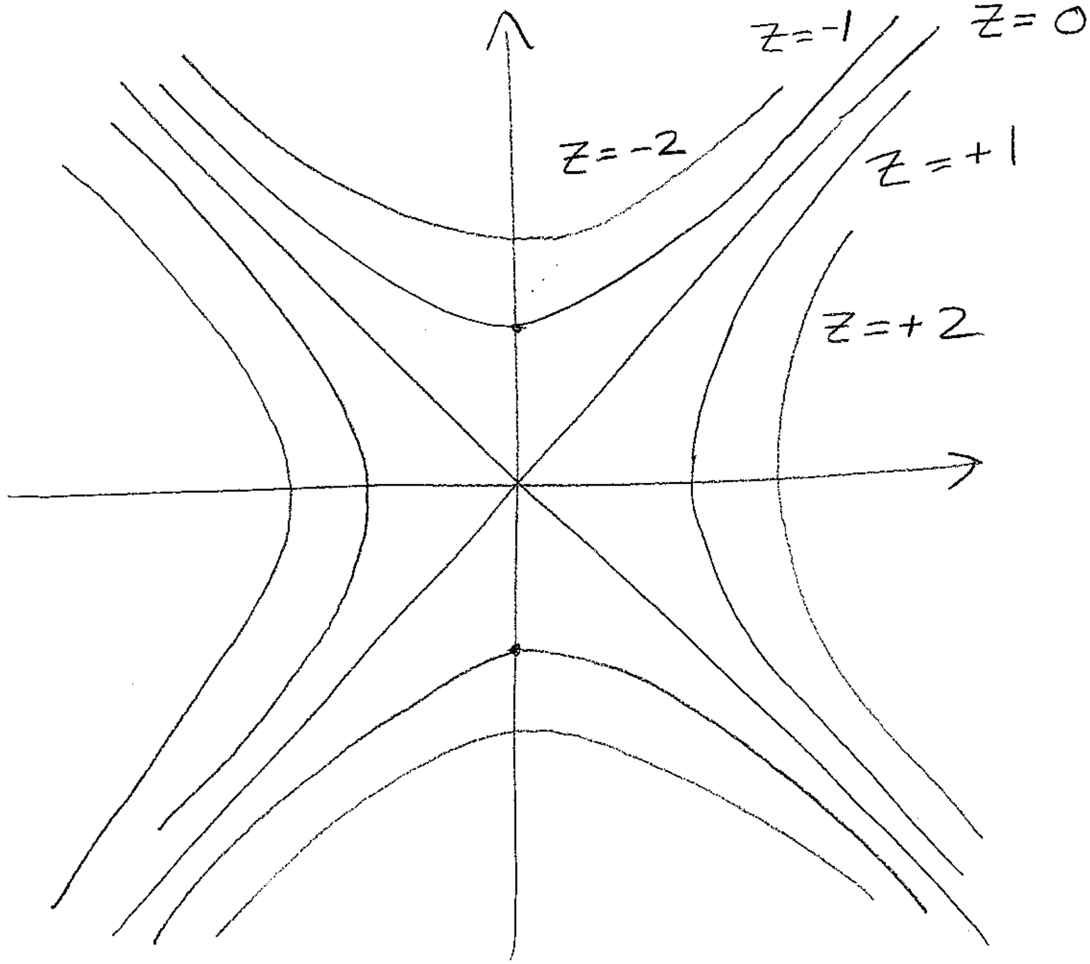
$\Longleftrightarrow x^2=y^2 \Leftrightarrow y=\pm x$

$z=-1$: $x^2-y^2=-1$

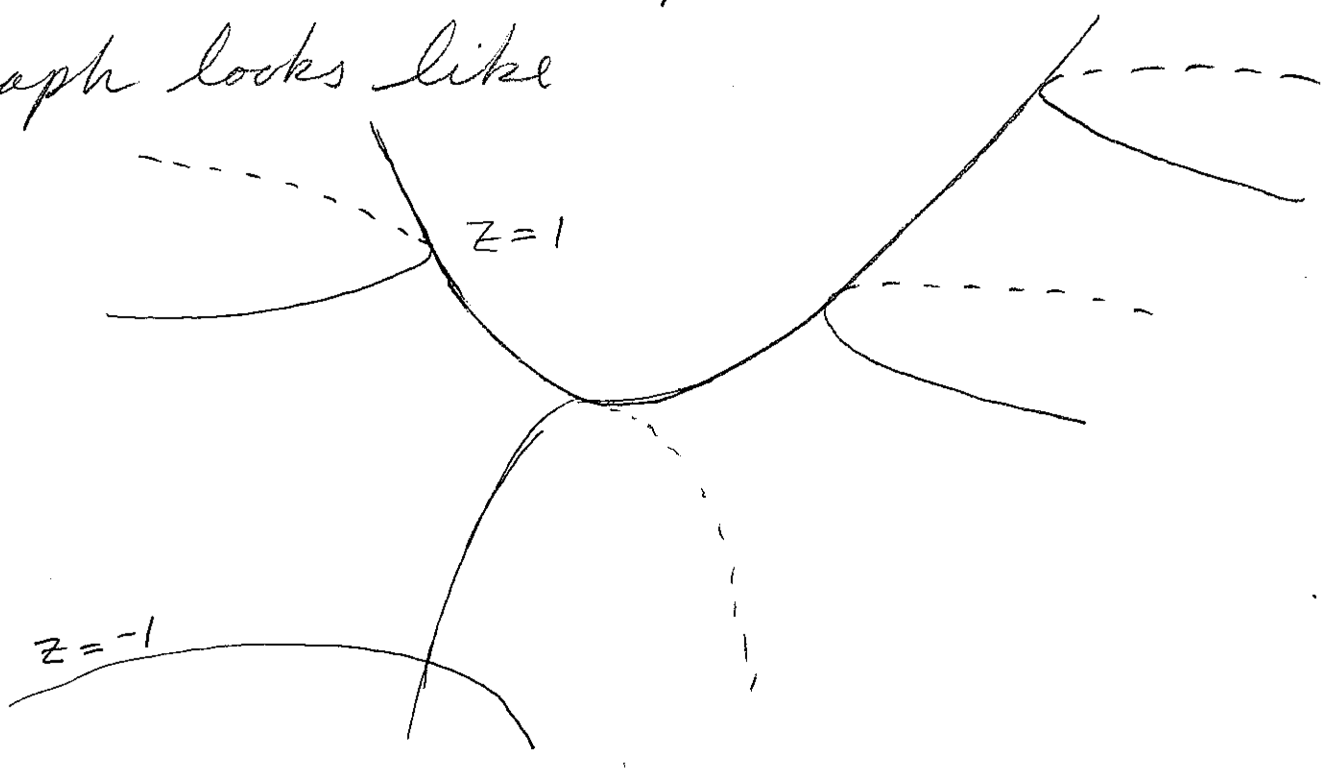
$\Longleftrightarrow y^2=x^2+1$

$z=+1$: $x^2-y^2=+1$

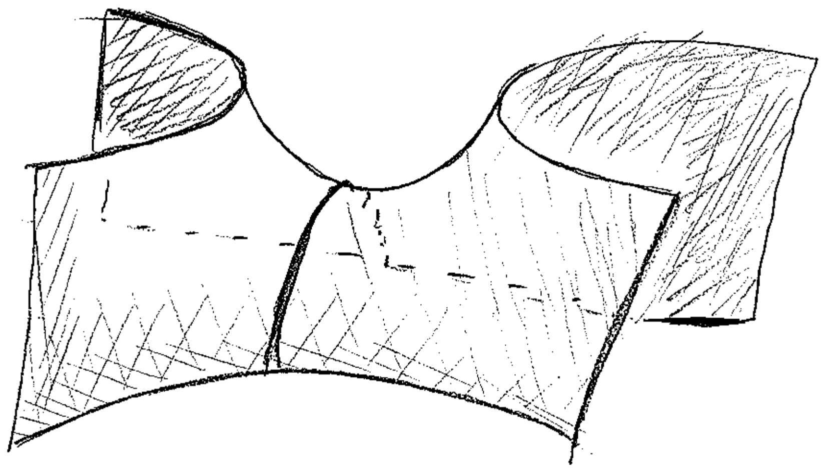
$\Longleftrightarrow x^2=y^2+1$



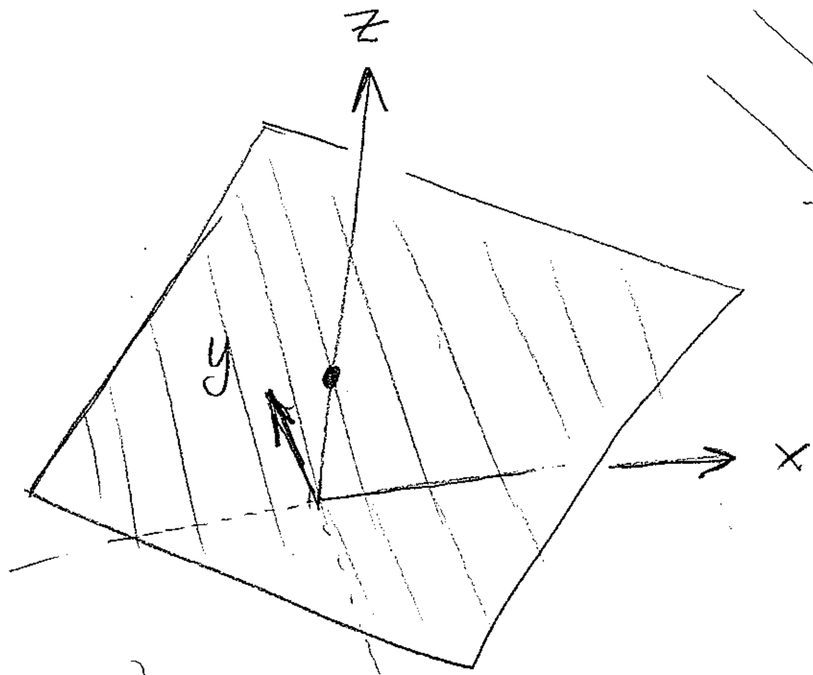
Each of these is a level set, like a contour line on a map. So, the graph looks like

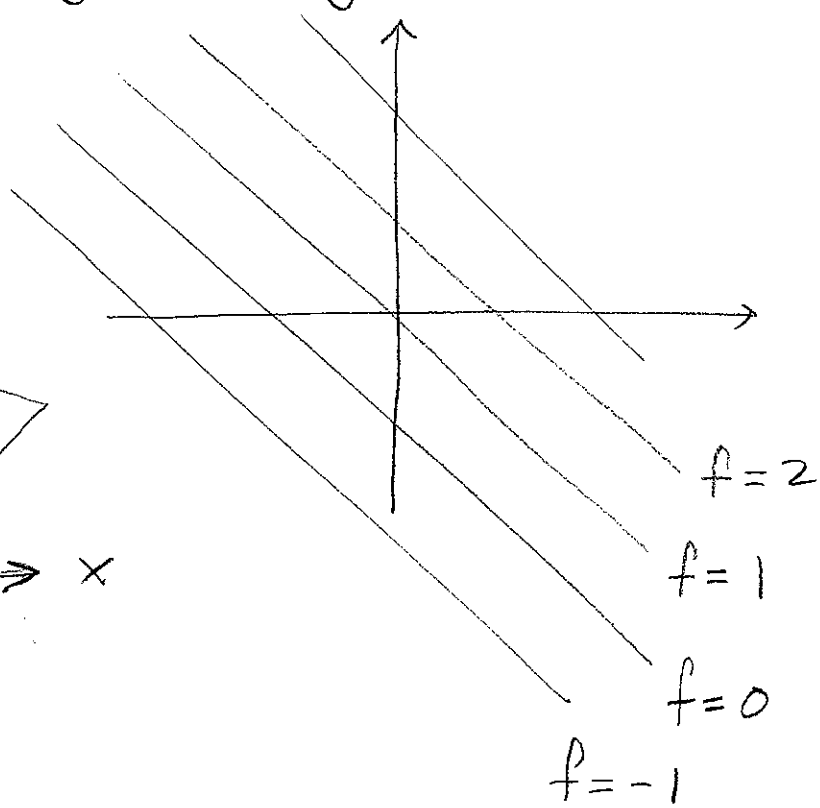


In other words, the graph is a saddle:



<u>Ex</u>: $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ $f(x,y) = x+y+1$





<u>Next time:</u> $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$, e.g.
$f(x,y,z) = x^2+y^2+z^2$. No graph (its in $\mathbb{R}^4$!)
but we do still have level sets.