2. The tip of the moving vector $\mathbf{r}(t)$ of a continuous vector function traces out a space curve.

3. The tangent vector to a smooth curve at a point P with position vector $\mathbf{r}(t)$ is the vector $\mathbf{r}'(t)$. The tangent line at P is the line through P parallel to the tangent vector $\mathbf{r}'(t)$. The unit tangent vector is $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$.

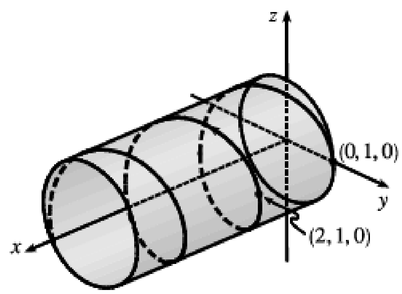
5. Use Formula 13.3.2, or equivalently, 13.3.3.

1. True. If we reparametrize the curve by replacing $u = t^3$, we have $\mathbf{r}(u) = u\,\mathbf{i} + 2u\,\mathbf{j} + 3u\,\mathbf{k}$, which is a line through the origin with direction vector $\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$.

2. True. Parametric equations for the curve are $x = 0$, $y = t^2$, $z = 4t$, and since $t = z/4$ we have $y = t^2 = (z/4)^2$ or $y = \frac{1}{16}z^2$, $x = 0$. This is an equation of a parabola in the yz-plane.

4. True. See Theorem 13.2.2.

1. (a) The corresponding parametric equations for the curve are $x = t$, $y = \cos\,\pi t,\ z = \sin\,\pi t$. Since $y^2 + z^2 = 1$, the curve is contained in a circular cylinder with axis the x-axis. Since $x = t$, the curve is a helix.



(b) $\mathbf{r}(t) = t\,\mathbf{i} + \cos\,\pi t\,\mathbf{j} + \sin\,\pi t\,\mathbf{k} \quad \Rightarrow$

$\mathbf{r}'(t) = \mathbf{i} - \pi\sin\,\pi t\,\mathbf{j} + \pi\cos\,\pi t\,\mathbf{k} \quad \Rightarrow$

$\mathbf{r}''(t) = -\pi^2\cos\,\pi t\,\mathbf{j} - \pi^2\sin\,\pi t\,\mathbf{k}$

2. (a) The expressions $\sqrt{2-t}$, $(e^t - 1)/t$, and $\ln(t+1)$ are all defined when $2 - t \geq 0 \quad \Rightarrow \quad t \leq 2, t \neq 0$, and $t + 1 > 0 \quad \Rightarrow \quad t > -1$. Thus the domain of $\mathbf{r}$ is $(-1, 0) \cup (0, 2]$.

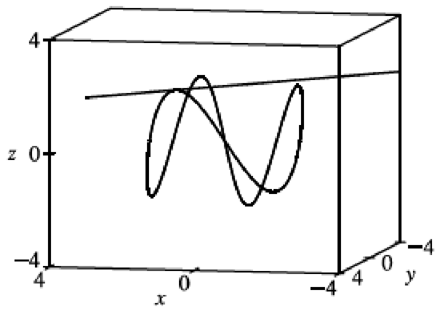
(b) $\displaystyle\lim_{t\to 0} \mathbf{r}(t) = \left\langle \lim_{t\to 0}\sqrt{2-t}, \lim_{t\to 0}\frac{e^t-1}{t}, \lim_{t\to 0}\ln(t+1) \right\rangle = \left\langle \sqrt{2-0}, \lim_{t\to 0}\frac{e^t}{1}, \ln(0+1) \right\rangle$

$= \langle\sqrt{2}, 1, 0\rangle$ [using l'Hospital's Rule in the y-component]

(c) $\displaystyle\mathbf{r}'(t) = \left\langle \frac{d}{dt}\sqrt{2-t}, \frac{d}{dt}\frac{e^t-1}{t}, \frac{d}{dt}\ln(t+1) \right\rangle = \left\langle -\frac{1}{2\sqrt{2-t}}, \frac{te^t - e^t + 1}{t^2}, \frac{1}{t+1} \right\rangle$

3. The projection of the curve C of intersection onto the xy-plane is the circle $x^2 + y^2 = 16$, $z = 0$. So we can write $x = 4\cos t,\ y = 4\sin t,\ 0 \leq t \leq 2\pi$. From the equation of the plane, we have $z = 5 - x = 5 - 4\cos t$, so parametric equations for C are $x = 4\cos t,\ y = 4\sin t,\ z = 5 - 4\cos t$, $0 \leq t \leq 2\pi$, and the corresponding vector function is $\mathbf{r}(t) = 4\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + (5 - 4\cos t)\,\mathbf{k}$, $0 \leq t \leq 2\pi$.

4. The curve is given by $\mathbf{r}(t) = \langle 2\sin t, 2\sin 2t, 2\sin 3t\rangle$, so $\mathbf{r}'(t) = \langle 2\cos t, 4\cos 2t, 6\cos 3t\rangle$. The point $(1, \sqrt{3}, 2)$ corresponds to $t = \frac{\pi}{6}$ (or $\frac{\pi}{6} + 2k\pi$, k an integer), so the tangent vector there is $\mathbf{r}'(\frac{\pi}{6}) = \langle\sqrt{3}, 2, 0\rangle$. Then the tangent line has direction vector $\langle\sqrt{3}, 2, 0\rangle$ and includes the point $(1, \sqrt{3}, 2)$, so parametric equations are $x = 1 + \sqrt{3}\,t$, $y = \sqrt{3} + 2t$, $z = 2$.



5. $\int_0^1 (t^2\,\mathbf{i} + t\cos\pi t\,\mathbf{j} + \sin\pi t\,\mathbf{k})\,dt = \left(\int_0^1 t^2\,dt\right)\mathbf{i} + \left(\int_0^1 t\cos\pi t\,dt\right)\mathbf{j} + \left(\int_0^1 \sin\pi t\,dt\right)\mathbf{k}$

$$= \left[\tfrac{1}{3}t^3\right]_0^1\,\mathbf{i} + \left(\tfrac{t}{\pi}\sin\pi t\Big]_0^1 - \int_0^1 \tfrac{1}{\pi}\sin\pi t\,dt\right)\mathbf{j} + \left[-\tfrac{1}{\pi}\cos\pi t\right]_0^1\,\mathbf{k}$$

$$= \tfrac{1}{3}\,\mathbf{i} + \left[\tfrac{1}{\pi^2}\cos\pi t\right]_0^1\,\mathbf{j} + \tfrac{2}{\pi}\,\mathbf{k} = \tfrac{1}{3}\,\mathbf{i} - \tfrac{2}{\pi^2}\,\mathbf{j} + \tfrac{2}{\pi}\,\mathbf{k}$$

where we integrated by parts in the y-component.

6. (a) C intersects the xz-plane where $y = 0 \;\Rightarrow\; 2t - 1 = 0 \;\Rightarrow\; t = \frac{1}{2}$, so the point is $\left(2 - \left(\frac{1}{2}\right)^3, 0, \ln\frac{1}{2}\right) = \left(\frac{15}{8}, 0, -\ln 2\right)$.

(b) The curve is given by $\mathbf{r}(t) = \langle 2 - t^3, 2t - 1, \ln t\rangle$, so $\mathbf{r}'(t) = \langle -3t^2, 2, 1/t\rangle$. The point $(1, 1, 0)$ corresponds to $t = 1$, so the tangent vector there is $\mathbf{r}'(1) = \langle -3, 2, 1\rangle$. Then the tangent line has direction vector $\langle -3, 2, 1\rangle$ and includes the point $(1, 1, 0)$, so parametric equations are $x = 1 - 3t$, $y = 1 + 2t$, $z = t$.

(c) The normal plane has normal vector $\mathbf{r}'(1) = \langle -3, 2, 1\rangle$ and equation $-3(x-1) + 2(y-1) + z = 0$ or $3x - 2y - z = 1$.

8. $\mathbf{r}'(t) = \left\langle 3t^{1/2}, -2\sin 2t, 2\cos 2t\right\rangle$, $\quad|\mathbf{r}'(t)| = \sqrt{9t + 4(\sin^2 2t + \cos^2 2t)} = \sqrt{9t + 4}$.

Thus $L = \int_0^1 \sqrt{9t + 4}\,dt = \int_4^{13} \frac{1}{9}u^{1/2}\,du = \frac{1}{9}\cdot\frac{2}{3}u^{3/2}\Big]_4^{13} = \frac{2}{27}(13^{3/2} - 8)$.

9. The angle of intersection of the two curves, θ, is the angle between their respective tangents at the point of intersection. For both curves the point $(1, 0, 0)$ occurs when $t = 0$.

$\mathbf{r}_1'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \mathbf{k} \;\Rightarrow\; \mathbf{r}_1'(0) = \mathbf{j} + \mathbf{k}$ and $\mathbf{r}_2'(t) = \mathbf{i} + 2t\,\mathbf{j} + 3t^2\,\mathbf{k} \;\Rightarrow\; \mathbf{r}_2'(0) = \mathbf{i}$.

$\mathbf{r}_1'(0)\cdot\mathbf{r}_2'(0) = (\mathbf{j} + \mathbf{k})\cdot\mathbf{i} = 0$. Therefore, the curves intersect in a right angle, that is, $\theta = \frac{\pi}{2}$.

10. The parametric value corresponding to the point $(1, \mathbf{0}, 1)$ is $t = \mathbf{0}$.

$\mathbf{r}'(t) = e^t\,\mathbf{i} + e^t(\cos t + \sin t)\,\mathbf{j} + e^t(\cos t - \sin t)\,\mathbf{k} \quad \Rightarrow \quad |\mathbf{r}'(t)| = e^t\sqrt{1 + (\cos t + \sin t)^2 + (\cos t - \sin t)^2} = \sqrt{3}\,e^t$

and $s(t) = \int_0^t e^u \sqrt{3}\;du = \sqrt{3}(e^t - 1) \quad \Rightarrow \quad t = \ln\!\left(1 + \frac{1}{\sqrt{3}}s\right)$.

Therefore, $\mathbf{r}(t(s)) = \left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{i} + \left(1 + \frac{1}{\sqrt{3}}s\right)\sin\ln\!\left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{j} + \left(1 + \frac{1}{\sqrt{3}}s\right)\cos\ln\!\left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{k}$.