1. (8 points) Find an equation for the plane that passes through the point $P = (1, 2, 3)$ and contains the line L given by the parametric equation

$$x(t) = 1 - 3t, \quad y(t) = 3, \quad \text{and} \quad z(t) = 6 + 2t$$

for $-\infty < t < \infty$.

$$\boxed{}\ x + \boxed{}\ y + \boxed{}\ z = \boxed{}$$

Find a normal vector to define the plane.

i.e. Find two vectors parallel to the plane and use Cross product.

Direction vector of the line $\vec{v} = \langle -3, 0, 2 \rangle$ is parallel to the plane.

Also, pick any point from the line, say $Q = (1, 3, 6)$.

then $\vec{PQ} = (1,3,6) - (1,2,3) = \langle 0, 1, 3 \rangle$ is also parallel to the plane.

$$\vec{n} = \vec{v} \times \vec{PQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 0 & 2 \\ 0 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} \vec{i} - \begin{vmatrix} -3 & 2 \\ 0 & 3 \end{vmatrix} \vec{j}$$

$$+ \begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} \vec{k} = -2\vec{i} + 9\vec{j} - 3\vec{k} = \langle -2, 9, -3 \rangle$$ is perpendicular to the plane.

There fore; equation for the plane is.

$$-2(x-1) + 9(y-2) - 3(z-3) = 0$$

$$\Leftrightarrow -2x + 9y - 3z = 7$$

<u>Solution</u>: $-2x + 9y - 3z = 7$.

2. (5 points) Find $\text{proj}_{\mathbf{a}}\,\mathbf{b}$, the vector projection of $\mathbf{b}$ onto $\mathbf{a}$, when $\mathbf{a} = \langle 1, 3, 2 \rangle$ and $\mathbf{b} = \langle 2, -1, 0 \rangle$.

$$\text{proj}_{\mathbf{a}}\,\mathbf{b} = \left\langle \boxed{}, \boxed{}, \boxed{} \right\rangle$$

$$\text{Proj}_{\vec{a}}\,\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\,\vec{a} = \frac{1*2 + 3*(-1) + 2*0}{1*1 + 3*3 + 2*2}\langle 1, 3, 2 \rangle$$

$$= -\frac{1}{14}\langle 1, 3, 2 \rangle$$

$$= \langle -\frac{1}{14}, -\frac{3}{14}, \frac{-2}{14} \rangle$$

Solution $\langle -\frac{1}{14}, -\frac{3}{14}, \frac{-2}{14} \rangle$

3. (4 points) Which statement is true in $\mathbb{R}^3$?

- [ ] Two planes perpendicular to a third plane are parallel.
- [ ] Two lines parallel to the same plane are parallel.
- [ ] Two lines either intersect or are parallel.
- [V] Two planes either intersect or are parallel.

1st statement : Counter example would be xy-plane, xz-plane, yz-plane.
2nd Statement : Counter example would be xy-plane, intersecting lines with same z
3rd Statement : Counter example would be skew lines.
4th statement : True.

4. (4 points) Mark exactly one box corresponding to the correct ending of the sentence.

$$\text{“ The limit} \quad \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+y^4} \quad \ldots$$

- [V] ... does not exist because the limits as one approaches $(0,0)$ along the lines $x = 0$ and $y = x$ are different.”
- [ ] ... does not exist because the limits as one approaches $(0,0)$ along the curves $y = x^2$ and $x = y^2$ are different.”
- [ ] ... exists because $\frac{x^2y^2}{x^4+y^4}$ is a composition of continuous functions”
- [ ] ... exists because the partial derivatives of $\frac{x^2y^2}{x^4+y^4}$ are continuous at $(0,0)$”
- [ ] ... exists because the limits as one approaches $(0,0)$ along the lines $y = x$ and $y = -x$ are the same.”

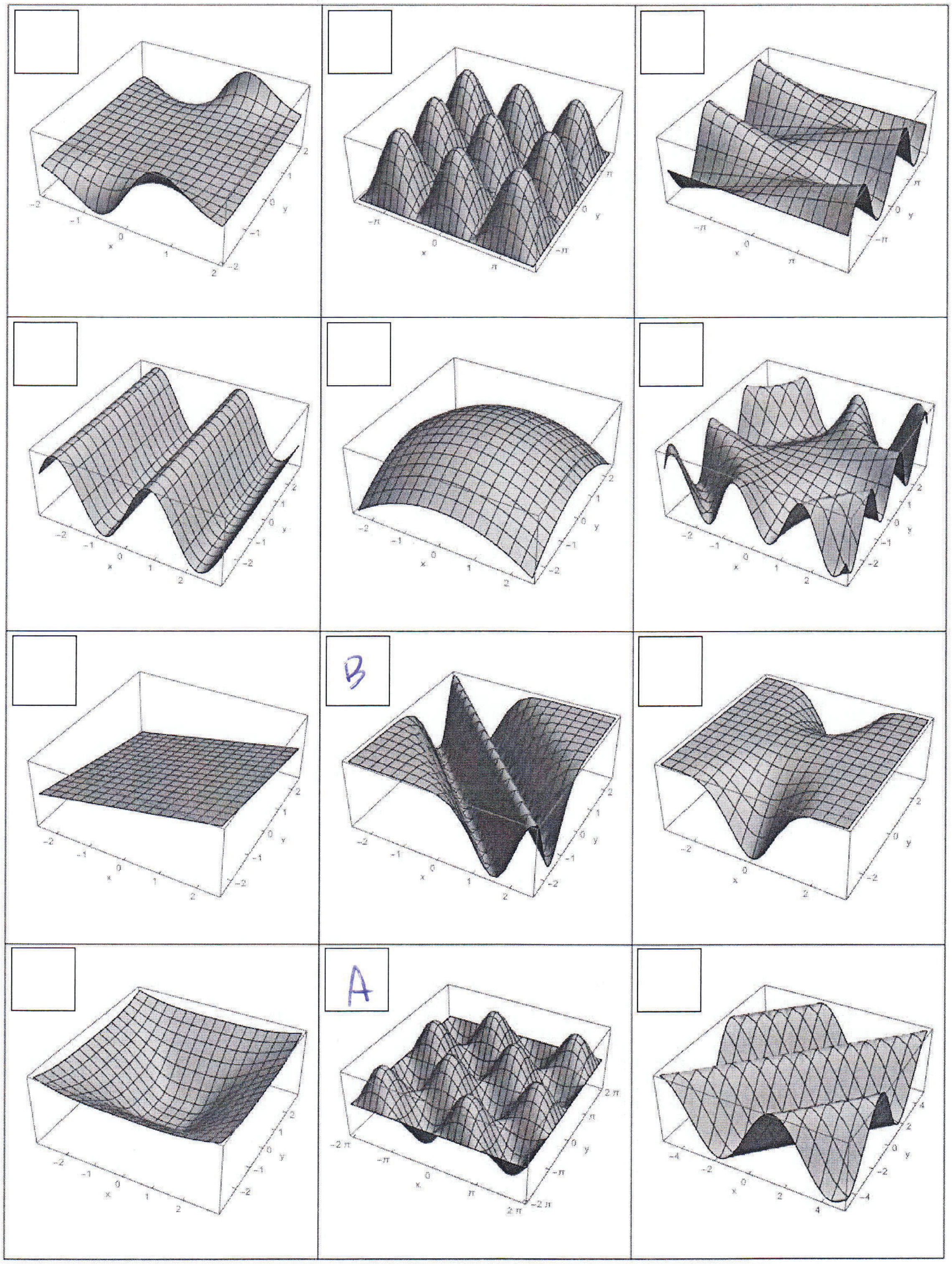
2nd statement : same limits along $y=x^2$, $x=y^2$.
3rd " : discontinuous at (0,0).
4th " : discontinuous at (0,0).
5th " : Limit exists but 1st statement says there exists a path that the limit does not exist.

(a) has value 0 at $x=n\pi$, $y=m\pi$ n, m are integers.
(b). has value 0 along $x+y=0$.
Put $x+y=k$, then the function becomes $-k^2e^{-k^2}$
• As $|k|\to\infty$, it goes to 0.
• Also, first derivative $-2ke^{-k^2}(1+k)(1-k)$ implies minimum at $k=\pm 1$

5. (6 points) For each function

(a) $\sin(x)\sin(y)$ (b) $-(x+y)^2e^{-(x+y)^2}$

label its graph from among the options below.



6. (6 points) Consider the function $f(x, y, z) = \cos(x) + x\sin(y) + y^2 z$.
Compute $f_x(\frac{\pi}{2}, 0, 0)$.

$$f_x\left(\frac{\pi}{2}, 0, 0\right) = \boxed{}$$

$$f_x(x,y,z)\Big|_{(\frac{\pi}{2},0,0)} = -\sin x + \sin y\Big|_{(\frac{\pi}{2},0,0)}$$
$$= -1$$

Solution -1

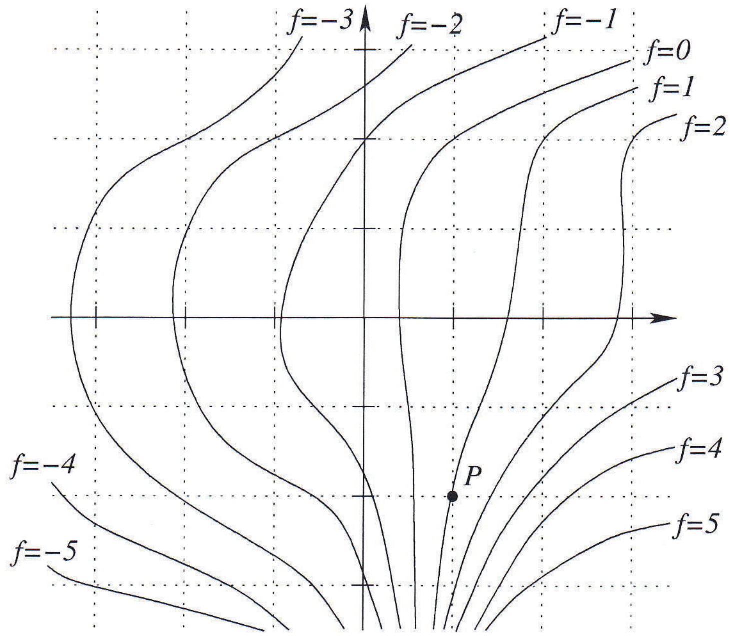
Compute $f_{zy}(0, \pi, 2)$.

$$f_{zy}(0, \pi, 2) = \boxed{}$$

$$f_z(x,y,z) = y^2$$
$$f_{zy}(x,y,z)\Big|_{(0,\pi,2)} = 2y\Big|_{(0,\pi,2)} = 2\pi$$

Solution 2π

7. (10 points) Consider the differentiable function f whose level curves (or contours) are shown in the figure. The points $(0,0)$ and $(1,0)$ are labeled for reference.



A. Circle the best answer. $f(2,2) =$ Read the contour curve at $(2,2)$

−3	−2	−1	0	1	2	3

B. Circle the best answer. $f_{xy}(1,-2)$ is

Note $f_{xy}(1,-2) = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f(1,-2)\right)$

we need to read $\frac{\partial}{\partial x} f$ depending on y at the point $(1,-2)$

positive	negative	zero

C. Circle the best estimate for $h'(0)$ where $h(t) = f(\sin(t), t^2 + 3t + 2)$.

step 1: Fix x at $(1,-2)$

Step 2: Read $\frac{\partial f}{\partial x}$ as y increases

−10	−5 or 0		5	10

By chain rule, $\frac{dh}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$.

$\frac{dh}{dt}\Big|_{t=0} \approx \frac{f(0+\delta, 2) - f(0,2)}{\delta} * (\cos t|_{t=0}) + \frac{f(0, 2+\delta) - f(0,2)}{\delta} * (2t+3|_{t=0})$

To approximate partial derivatives, choose small enough δ.

For example, if $\delta = 1$ $\frac{dh}{dt}\Big|_{t=0} \approx \frac{0+1}{1} \cdot 1 + \frac{-2.5+1}{1} \cdot 3 = -3.5$

8. (7 points) Find the equation of the tangent plane to the graph $z = x^3 - 2\cos(y)$ at the point $(1, 0, -1)$.

$\boxed{}\ x + \boxed{}\ y + \boxed{}\ z = \boxed{}$

i) $f(1,0) = -1$

ii) $f_x|_{(1,0)} = 3x^2|_{(1,0)} = 3$

iii) $f_y|_{(1,0)} = 2\sin y|_{(1,0)} = 0$

Therefore, the equation for plane.

$z = -1 + 3(x-1) + 0 \cdot (y-0)$

$\Leftrightarrow \quad 3x + 0y - z = 4$

Solution $\quad 3x + 0 \cdot y - z = 4$