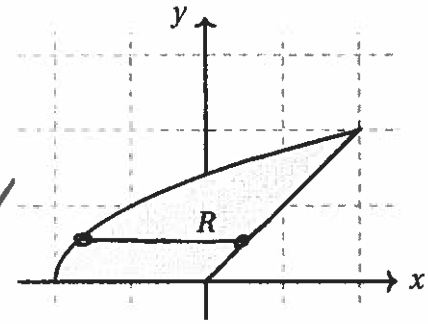
1. Let R be the region shown which is bounded by the curve $y^2 - x - 2 = 0$, the line $y = x$, and x-axis. Evaluate $\iint_R 3y\,dA$. **(4 points)**



$$\int_0^2 \int_{y^2-2}^{y} 3y\,dx\,dy = \int_0^2 3y(y+2-y^2)\,dy$$

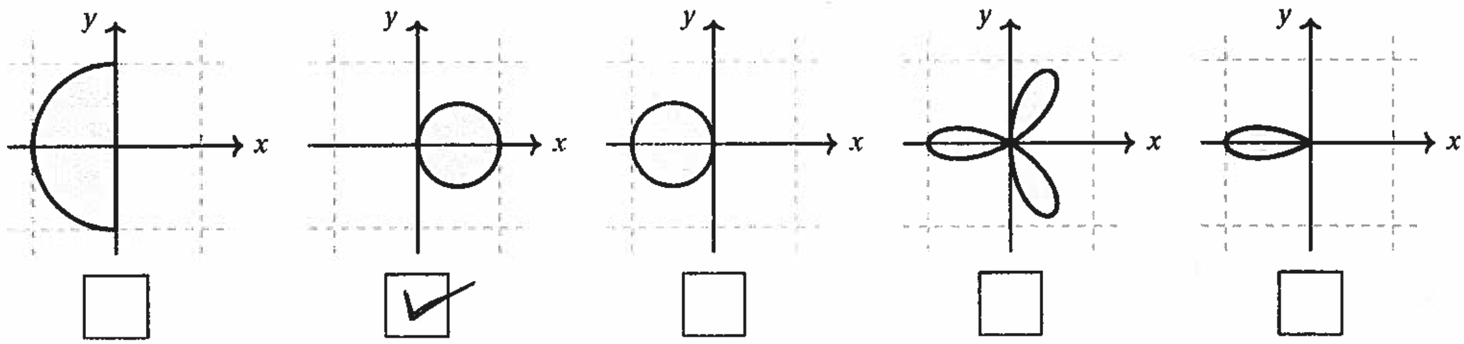
$$= \int_0^2 3y^2+6y-3y^3\,dy = y^3+3y^2-\frac{3}{4}y^4 \Big|_{y=0}^{y=2}$$

$$= 8+12-3\cdot\frac{16}{4} = 8$$

$$\boxed{\iint_R 3y\,dA = 8}$$

2. The integral $\iint_R 2x^2 + 2y^2 + y\,dA$ has the form $\int_{-\pi/2}^{\pi/2}\int_0^{\cos\theta} ??\,dr\,d\theta$ when converted into polar coordinates.

(a) Mark the box below the picture of the region that represents R. **(1 point)**



(b) Fill in the missing integrand to convert this integral into polar coordinates. **(2 points)**

$$\iint_R 2x^2 + 2y^2 + y\,dA = \int_{-\pi/2}^{\pi/2}\int_0^{\cos\theta} \boxed{(2r^2+r\sin\theta)\,r}\,dr\,d\theta.$$

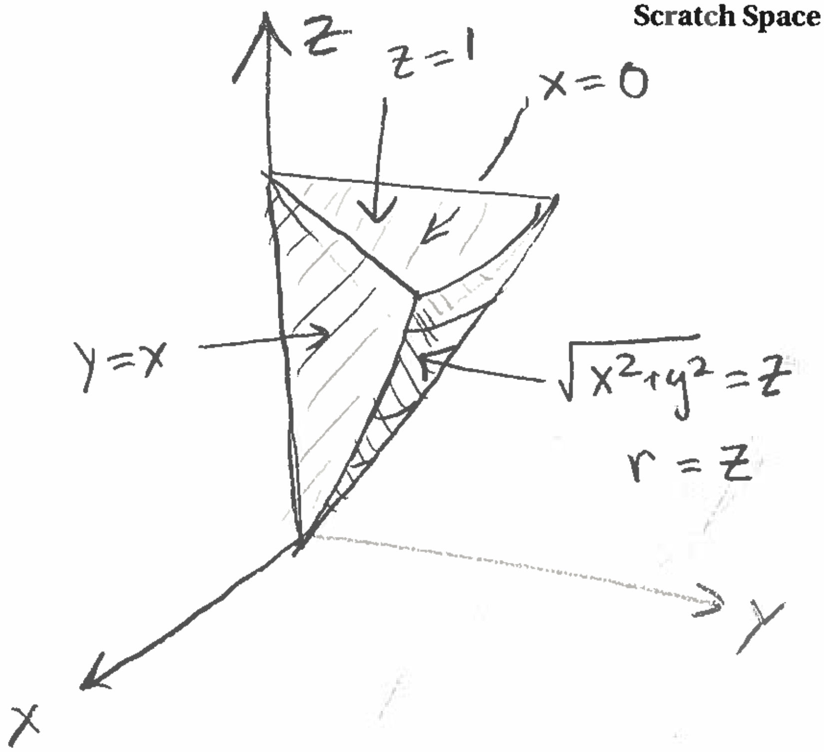
Scratch Space

3. Consider the region R in the positive octant bounded by the cone $z = \sqrt{x^2 + y^2}$ and the planes $z = 1$, $x = 0$, and $y = x$. In **each column** below, exactly one of the iterated integrals computes $\iiint_R x\,dV$. Determine which are the correct answers and mark the boxes next to them. **(4 points)**

- [ ] $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sec\phi} \rho^3 \sin^2\phi \cos\theta \, d\rho \, d\phi \, d\theta$
- [x] $\int_{\pi/4}^{\pi/2} \int_0^{\pi/4} \int_0^{\sec\phi} \rho^3 \sin^2\phi \cos\theta \, d\rho \, d\phi \, d\theta$
- [ ] $\int_{\pi/4}^{\pi/2} \int_0^{\pi/4} \int_0^{\sec\phi} \rho^2 \sin^2\phi \cos\theta \, d\rho \, d\phi \, d\theta$

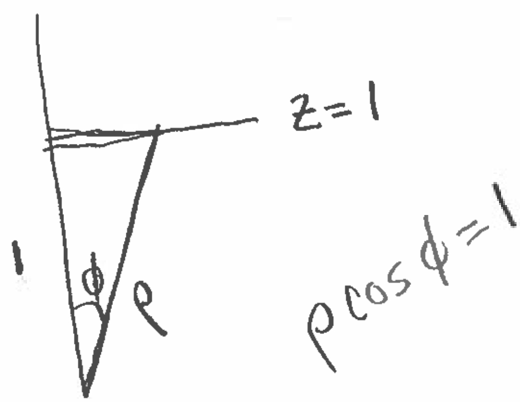
- [ ] $\int_0^1 \int_{\pi/4}^{\pi/2} \int_0^{z^2} r^2 \cos\theta \, dr \, d\theta \, dz$
- [ ] $\int_0^1 \int_{\pi/4}^{\pi/2} \int_0^{z} r \cos\theta \, dr \, d\theta \, dz$
- [x] $\int_0^1 \int_{\pi/4}^{\pi/2} \int_0^{z} r^2 \cos\theta \, dr \, d\theta \, dz$

Scratch Space



Cylindrical:

$$\int_0^1 \int_{\pi/4}^{\pi/2} \int_0^{z} r^2 \cos\theta \quad d\theta \, dz$$

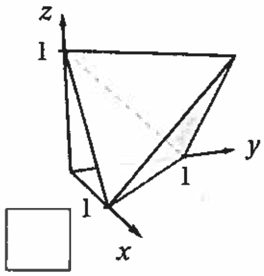
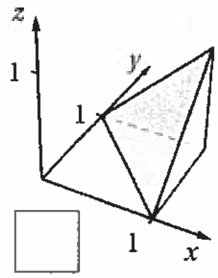
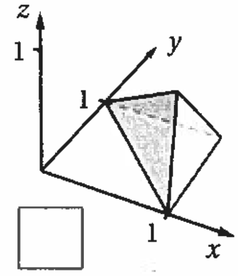
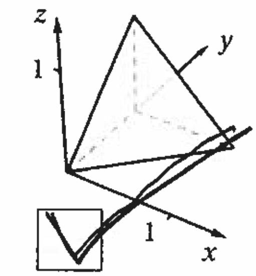
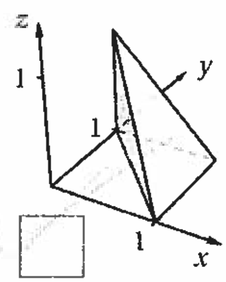
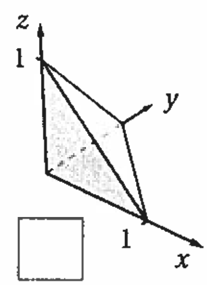


4. A rectangular metallic plate R is placed in the plane with vertices at $(-2,-1)$, $(-2,1)$, $(2,-1)$, and $(2,1)$. The density (in g/cm^2) of the plate, $\rho(x,y)$, at various points is shown in the table, where x and y are measured in cm. Circle the best estimate for the mass of the plate. **(2 points)**

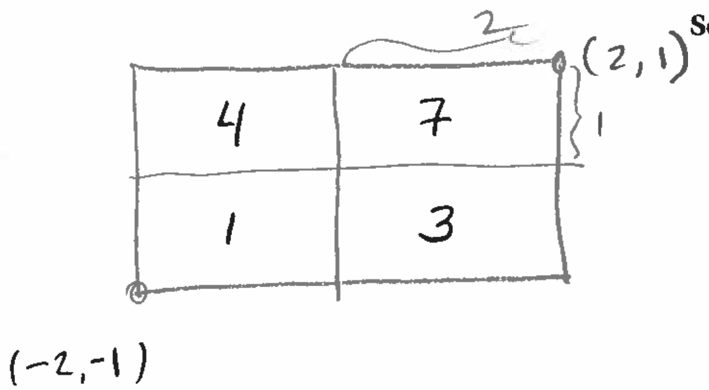
	$\rho(x,y)$	$x = -1$	$x = 1$
y	$1/2$	4	7
	$-1/2$	1	3

Mass of $R \approx$	0	4	15	30	46	60	78	grams.

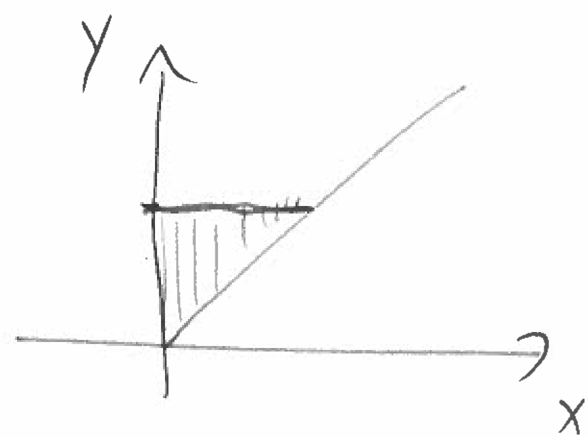
5. The integral of the function $f(x,y,z) = 2x$ over a region R is computed by $\int_0^1 \int_0^y \int_0^{y-x} 2x \, dz dx dy$. Mark the box below the picture of the region R. **(2 points)**

Scratch Space



15×2



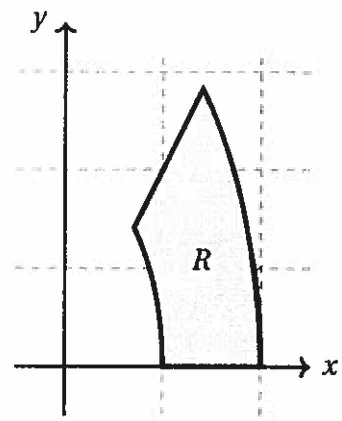
$\int_0^1$

6. Suppose R is the region in the first quadrant between the ellipses $x^2 + \frac{y^2}{4} = 1$ and $x^2 + \frac{y^2}{4} = 4$ and the lines $y = 0$ and $y = 2x$ shown at the right. Using the transformation

$$T(u, v) = \langle u\cos(v),\ 2u\sin(v)\rangle$$

$\cancel{1/\sqrt{2},\ 2\ 1/\sqrt{2}}$

find the integrand and limits of integration expressing the integral $\iint_R x\,dA$ as an iterated integral over a subset S in the uv–plane with $T(S) = R$. **(5 points)**



$$\int_1^2 \int_0^{\pi/4} (u\cos v)(2u)\, dv\, du$$

$$\iint_R x\,dA = \boxed{\int_0^{\pi/4}\int_1^2 (u\cos v)\ 2u\ du\,dv}$$

Note: The order of integration is already determined.

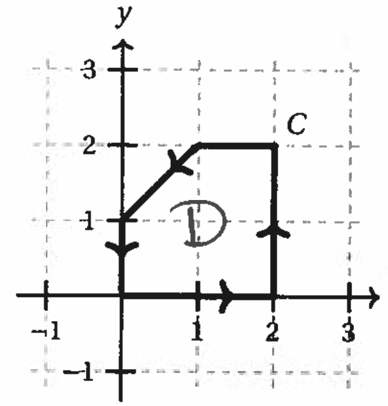
——— scratch ———

7. Compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = \langle \overset{P}{y+2\cos(x)}, \overset{Q}{3x+e^{y^2}} \rangle$ and C is the oriented curve shown. **(5 points)**

$$\int_C \vec{F}\cdot dr = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$$

$$= \iint_D 3-1 \, dA = 2\,\mathrm{Area}(D)$$

$$= 2\,(7/2) = 7$$

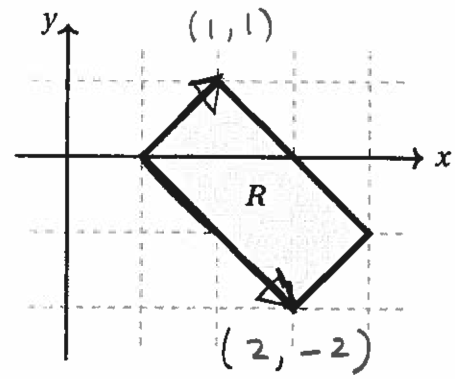


$\int_C \mathbf{F}\cdot d\mathbf{r} = \boxed{+7}$

8. Let R be the rectangle whose vertices are $(1,0)$, $(2,1)$, $(3,-2)$, and $(4,-1)$ shown at the right.

(a) Exactly one of the following defines a transformation $T(u,v)$ from the uv–plane to the xy–plane with $T(S) = R$, where $S = \{(u,v) \mid 0 \le u \le 1, 0 \le v \le 1\}$. Circle the correct formula for $T(u,v)$. **(2 points)**

$\langle 2u+3v,\ u-2v\rangle$	$\langle 2u+4v,\ u-v\rangle$	$\langle 2u+4v+1,\ u-v\rangle$
$\langle 2u+3v+1,\ u-2v\rangle$	$\langle u+2v,\ u-2v\rangle$	$\langle u+2v+1,\ u-2v\rangle$



4   -1

(b) $\iint_R y\,dA$ is negative zero positive **(1 point)**

Scratch Space

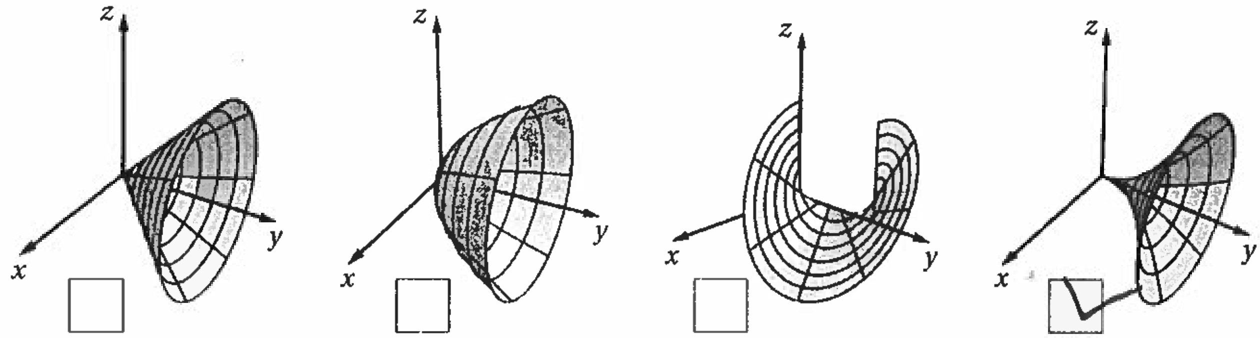
$$A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$$

$$(u,v) \longmapsto A\begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \langle u+2v+1,\ u-2v \rangle$$

9. Consider the surface S parameterized by $\mathbf{r}(u,v) = \langle u^2 \sin v,\ u,\ u^2 \cos v \rangle$ for $0 \le u \le 1$ and $0 \le v \le 2\pi$.

(a) Mark the box below the best picture of S. **(1 point)**



(b) Circle the correct formula for $\mathbf{r}_u \times \mathbf{r}_v$. **(2 points)**

$\langle u^2 \sin v,\ u^3,\ u^2 \cos v\rangle$	$\langle u\cos v,\ u^2,\ u\sin v\rangle$	$\langle -u^2 \sin v,\ 2u^3,\ -u^2 \cos v\rangle$	$\langle -u\cos v,\ 2u^2,\ -u\sin v\rangle$

(c) Circle the integrand for the integral $\int_0^1 \int_0^{2\pi} g(u,v)\, dv\, du$ that computes the surface area of S. **(2 points)**

$g(u,v) =$						
	$\sqrt{u^4+u^6}$	$\sqrt{u^2+u^4}$	$\sqrt{4u^4+4u^6}$	$\sqrt{4u^2+4u^4}$	$\sqrt{u^4+4u^6}$	$\sqrt{u^2+4u^4}$

(d) $\iint_S xz\, dS$ is | negative | zero | positive | **(1 point)**

Scratch Space

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} i & j & k \\ 2u\sin v & 1 & 2u\cos v \\ u^2\cos v & 0 & -u^2\sin v \end{vmatrix}$$

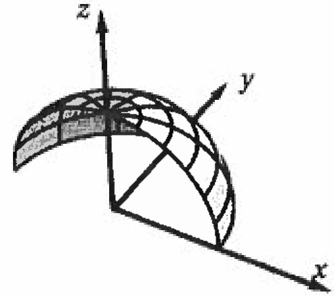
$$= \langle -u^2 \sin v,\ 2u^3,\ -u^2\cos v \rangle$$

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{u^4\sin^2 v + 4u^6 + u^4\cos^2 v}$$

$$= \sqrt{u^4 + 4u^6}$$

10. Parameterize each of the surfaces below with a function $\mathbf{r}(u, v)$. Be sure to specify the domain D of your parameterization.

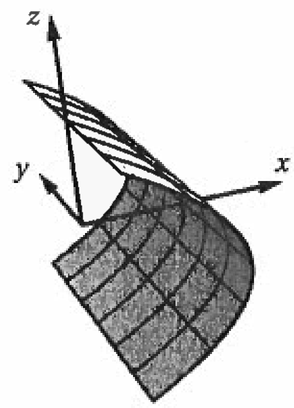
(a) The portion of the sphere $x^2 + y^2 + z^2 = 4$ where $y \geq 0$ and $z \geq 0$. **(3 points)**



$\mathbf{r}(u, v) = \langle 2 \sin u \cos v, 2 \sin u \sin v, 2 \cos u \rangle$

$D = \{(u, v) \mid 0 \leq u \leq \pi/2, \ 0 \leq v \leq \pi \}$

(b) The part of the graph $x = 1 - z^2$ where $x \geq 0$ and $-2 \leq y \leq 2$. **(4 points)**



$\mathbf{r}(u, v) = \langle 1 - v^2, u, v \rangle$

$D = \{(u, v) \mid -2 \leq u \leq 2, \ -1 \leq v \leq 1 \}$