Math 518: HW 2 due Wednesday, September 10, 2014.

1. This problem deals with constructing lots of smooth functions on manifolds. It consists of special cases of the results on pages 40-47 of Lee which we are skipping for now but will return to later in the semester.

   (a) Read Lemmas 2.20 and 2.21 of Lee and convince yourself there is a *smooth* function $f \colon \mathbb{R} \to [0,1]$ which is 1 on $[-1,1]$ and 0 outside $(-2,2)$.

   (b) Let M be a smooth n–manifold. Suppose (U,ϕ) is a smooth chart about a point $p \in M$ and set $a = \phi(p)$. As usual, let $B_r(a) = \{x \in \mathbb{R}^n \mid |x-a| < r\}$ be the Euclidean ball about a of radius r and $\bar{B}_r(a) = \{x \in \mathbb{R}^n \mid |x-a| \leq r\}$ be its closure. Suppose $r > 0$ is such that $\bar{B}_r(a) \subset \phi(U)$. For any $0 < s < r$, prove there is a smooth function on M which is 1 on $\phi^{-1}(\bar{B}_s(a))$ and 0 outside of $\phi^{-1}(B_r(a))$.

   (c) With the same notation as (b), suppose $f \colon \mathbb{R}^n \to \mathbb{R}$ is smooth. Show there exists a smooth function $\overline{f} \colon M \to \mathbb{R}$ which is equal to $f \circ \phi$ on some open neighborhood of p.

2. Problem 2-5 of Lee on page 48.

3. Problem 2-8 of Lee on page 48; do the first half about $\mathbb{R}\mathrm{P}^n$ only.

4. Prove Proposition 3.14 of Lee on page 59 in the case when $k = 2$.

5. Problem 3-6 of Lee on page 75.