Math 518: HW 5 due Wednesday, October 1, 2014.

1. Problem 8-3 of Lee on page 199. Do only the case of a smooth manifold without boundary.

2. Problem 8-13 of Lee on page 201.

3. Problem 7-2 of Lee on page 171.

4. Problem 7-6 of Lee on page 172.

5. **Problem corrected Sept 25.** Consider

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \;\middle|\; a \in \mathbb{R}^\times \quad \text{and} \quad b \in \mathbb{R} \right\}$$

   One way to think of G is as the group of affine transformations of $\mathbb{R}$, that is, diffeomorphisms of the form $x \mapsto ax + b$.

   (a) Prove that G with the operation of matrix multiplication is a Lie group.

   (b) Find an explicit formula for the left invariant vector field X on G where

$$X_e = \left. \frac{\partial}{\partial a} \right|_e + \left. \frac{\partial}{\partial b} \right|_e$$

   (c) Do the same for the right invariant vector field with the same value of X_e. Is it the same as your answer in (b)?

   (d) Using the identification of G to $\mathbb{R}^2$ with the y axis removed, draw pictures of the your vector fields in (b) and (c).

6. Problem 7-16 of Lee on page 172.