Math 416: HW 5 due Friday, March 4, 2016.

Webpage: http://dunfield.info/416

Wednesday's lecture: Prof. Chris Leininger will be substituting for me on Wednesday, March 2 as I will be out of town then.

Office hours: Due to my travels, my office hours this week will beThursday 3:30–5:00, and by appointment on Friday. My office is 378 Altgeld Hall.

Textbook: In the assignment, the main text is abbreviated as follows:

[FIS] Freidberg, Insel, Spence, *Linear Algebra*, 4th edition, 2002.

Problems:

1. Section 2.3 of [FIS], Problem 1.
2. Section 2.3 of [FIS], Problem 2.
3. Give matrices $A, B \in M_{2\times 2}(\mathbb{R})$ where AB is the zero matrix but BA is not.
4. Section 2.3 of [FIS], Problem 3.
5. Section 2.3 of [FIS], Problem 4 (a, b).
6. Section 2.4 of [FIS], Problem 1.
7. Suppose A and B are invertible $n \times n$ matrices.
   (a) Prove that $(AB)^{-1} = B^{-1}A^{-1}$.
   (b) Prove that $(A^t)^{-1} = (A^{-1})^t$
8. Prove Theorem 2.21 of [FIS]. This shows that any finite dimensional vector space V of dimension n is isomorphic to $\mathbb{R}^n$.
9. (a) Let A and B be $n \times n$ matrices such that AB is invertible. Prove that both A and B are invertible.
   (b) Give an example of two noninvertible matrices whose product is invertible.
   (c) Prove or give a counterexample: If A and B are nonzero $n \times n$ matrices with AB the zero matrix then A is not invertible.
10. Find the inverse of the following matrix, and check your answer two different ways.

$$A = \begin{pmatrix} 4 & -2 & -5 \\ -4 & 1 & 4 \\ -3 & 1 & 3 \end{pmatrix}$$