# Lecture 16: The HIKMOT method

<u>Recall</u>: $\mathbb{IR} = \{ \underline{x} = [x_0, x_1] \mid x_0, x_1 \in \mathbb{Q} \} := \underline{\mathbb{R}}$

Has arith ops, <u>interval extensions</u> of $f : \mathbb{R} \to \mathbb{R}$.

—— ∘ ——

Suppose $F : \mathbb{R}^n \to \mathbb{R}^n$ is $C^1$ and we seek $x_0$ with $F(x_0) = 0$.

[<u>Goal</u>: Turn $|F(x_1)|$ very small into $\exists$ such $x_0$ near $x_1$.]

<u>Newton's Method</u>: $N(x) = x - dF(x)^{-1} \cdot F(x)$

for $x \in \mathcal{U}$ where $dF$ is nonsing. Then $F(x_0) = 0$ $\iff N(x_0) = x_0$. [Moreover, such $x_0$ is an attract. f.p. of $N$; see pf of the inverse fn thm.] Same is true for $N_m(x) = x - C\, F(x)$ where $C$ is any invert. $n \times n$ matrix. [If $C \approx dF(x_0)^{-1}$ where $F(x_0) = 0$ then $N_m$ still has $x_0$ as an attract. fixed pt.]

<u>Goal</u>: Find a clsd ball $B$ with $N_m(B) \subseteq B$ and apply the Brouwer Fixed Pt Thm.

<u>Krawczyk Method</u>: ① $[x] \in \underline{\mathbb{R}}^n$ [product of intervals] — alt not for $\underline{x}$

② $x_0 \in [x]$ ③ $C$ $n \times n$ invert.

$$K(x_0, [x], F) := x_0 - C[F(x_0)] + \left(I - C\left[dF([x])\right]\right)([x] - x_0)$$

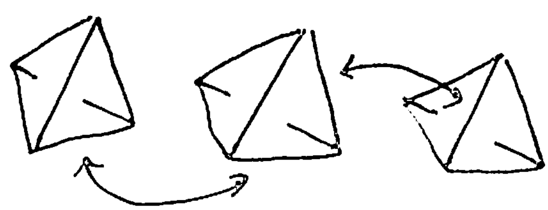
**Thm:** If $K(x_0, [x], F) \subseteq \operatorname{int}[x]$, there exists exactly one $x_1 \in [x]$ with $F(x_1) = 0$.

**Point:** $N_m([x]) \subseteq N_m(x_0) + \underbrace{[dN_m([x])]}_{\text{a rect box, so MVT is being applied coorwise.}} \cdot ([x] - x_0)$

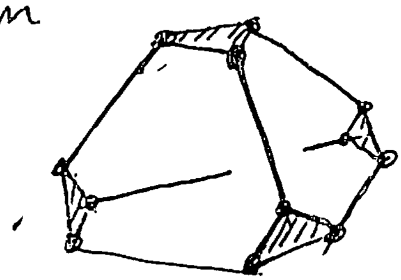
$\subseteq K(x_0, [x], F)$

**Pf of thm** (exist. only) Have $N_m([x]) \subseteq K(x_0, [x], F) \subseteq \operatorname{int}[x]$ so $N_m$ has a fixed pt in $[x] \Rightarrow \exists\, x_1 \in [x]$ with $F(x_1) = 0$. ▨

**Counting Gluing Equations:** $\mathcal{T}$ ideal tri. of $M^3$ with one cusp.



Now $\partial M$ is a torus with a triang coming from



$\chi(\mathcal{T}) = V - E + F - T$

$\chi(t^e) = v - e + f$

$\triangle$

$t^e$ = tri of $\partial M$. $\quad 0 = v - e + f = v - \frac{1}{2} f$

$\Rightarrow v = \frac{1}{2} f \Rightarrow 2E = \frac{1}{2}(4T) \Rightarrow E = T$

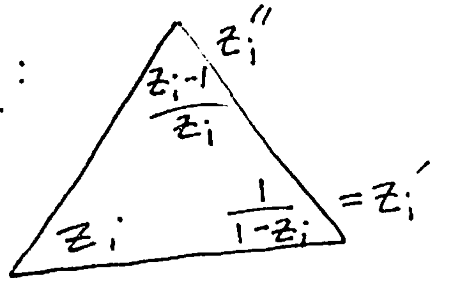
So we have $T$ vars ($z_i$ shapes)

but $T+2$ eqns ($T$ edge eqn + 2 cusp eqn)

Naively, expect $V(J) = \emptyset$ and can't apply K's method. Only gets worse with more cusps.

<u>Neumann-Zagier</u>: If $M$ has $n$ tet and $k$ cusps, then only $n-k$ eqns are needed.

<u>Recall</u>:




<u>Edge eqn</u>:

$$z_1 z_3' z_5'' z_6 = 1$$

$$z_1 \cdot \frac{1}{1-z_3} \cdot \frac{z_5 - 1}{z_5} \cdot z_6 = 1$$

$$z_1 (1-z_3)^{-1} z_5 (1-z_5)^{-1} z_6 = -1$$

<u>Gen form</u>: $\prod z_j^{a_{ij}} (1-z_j)^{b_{ij}} = (-1)^{e_i}$

NZ study the rank of $(A \mid B)$

where $A = (a_{ij})$ $B = (b_{ij})$ $\boxed{\text{DEMO!}}$