Lecture the last: Experiment and understanding.

So far: Can compute much about (hyperbolic) M^3.

< 1970: Isolated examples of hyp 3-mflds
1) Coxeter groups. 2) Seifert-Weber Dodec. sp.
3) Bianchi groups.

1970s:

Jørgensen: $S^3 \setminus$  is hyperbolic (fibers over S^1 with fiber )

Riley: Finds hyp. strs on simple knots by looking for $\pi_1 M \to PSL_2\mathbb{C}$, look for Dirichlet dom.

Thurston: Develops modern geom. ~~persp~~ perspective on M^3.

1980s:

Weeks: SnapPea
↳ Conjectures about str of hyp 3-mflds.
↳ Conjectures about Dehn filling. [Hodgson]

[Call back to Tait flyping conjecture..]

Roles for computation:

A) (Counter)examples. [Failure leads to evidence for conj.]

B) New conjectures/patterns.

C) Computer-aided proofs.

D) Doodling.

E) Computer inspired proofs.

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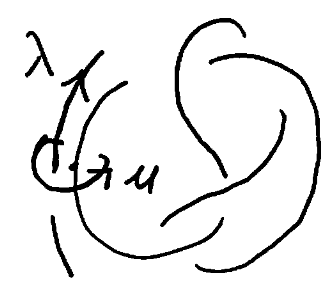
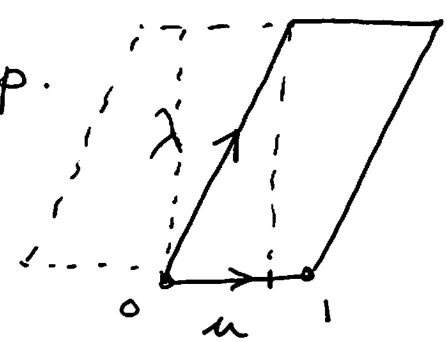
[Davies-Juhász-Lackenby-Tomasev] Posted < 10 days ago.

$K : S^1 \hookrightarrow S^3$ with $M = S^3 \setminus \mathring{N}(K)$ hyperbolic.

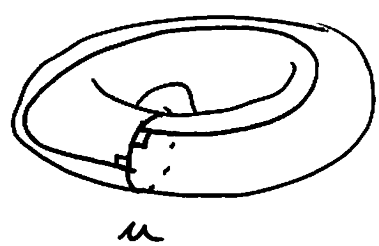
Basis for $H_1(\partial M; \mathbb{Z}) = \langle \mu, \lambda \rangle$
with λ char as the ker of
$H_1(\partial M) \longrightarrow H_1(M)$.



Look at torus cross-sec of cusp.
Up to scale looks like, set



$\text{slope}(K) = \mathrm{Re}(\lambda)$
$= \mathrm{Re}(\lambda/\mu)$
if unnormalized



Knot signature: $\sigma(K) \in \mathbb{Z}$ signature of a quadratic form assoc to a Seifert surface

$=$ sig of a 4-mfld with $\partial = \Sigma_2(K)$

K bounds a disc in $D^4 \Rightarrow \sigma(K) = 0$.

<u>Thm</u>: $\exists\, c_1 > 0$ such that $\forall$ hyp K,

$$|2\sigma(K) - \text{slope}(K)| \le c_1 \,\text{vol}(K)\, \text{inj}(K)^{-3}$$

where

$$\text{inj}(K) = \inf\{\text{inj}_x \mid x \in M \setminus (\text{max cusp})\}$$

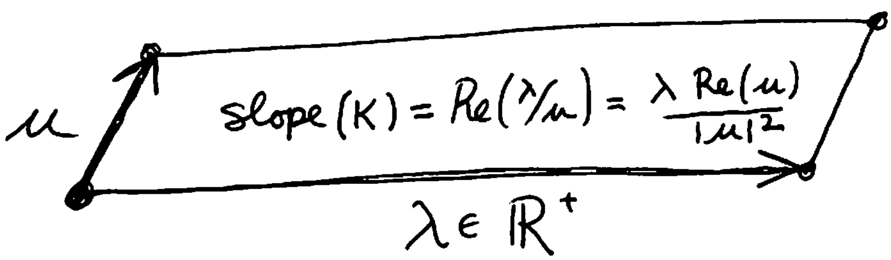
<u>Conj</u>: $c_1 = 0.3$

<u>Notes</u>: Goal was find rel. between hyp inv. and more topological ones. [After Floer theory?]

The idea of slope(K) came from data

[Fig in Nature paper sim. to Fig 2]

Looked at <u>max</u> cusp normalized like this:



Plot suggests $\sigma \approx c \cdot \lambda\, \text{Re}(\mu)$

<u>Note</u>: In max cusp, $1 \le |\mu| \le 6$.

[Show second plot.]

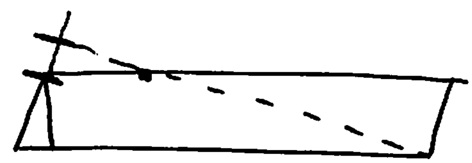
<u>Thm</u>: $\exists\, c_1 > 0$ such that for all hyp K

$$|2\sigma(K) - \text{slope}(K)| \le c_1 \, \text{vol}(K)\, \text{inj}(K)^{-3}$$

where

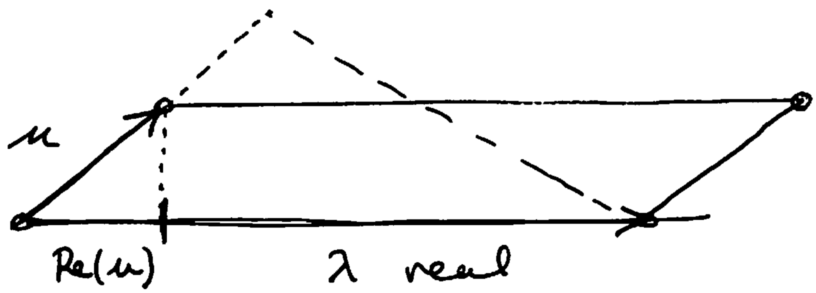
$$\text{inj}(K) = \inf \{ \text{inj}_x \mid x \in M \setminus (\text{max cusp}) \}$$

<u>Conj</u>: $c_1 = 0.3$



or version in Nature.

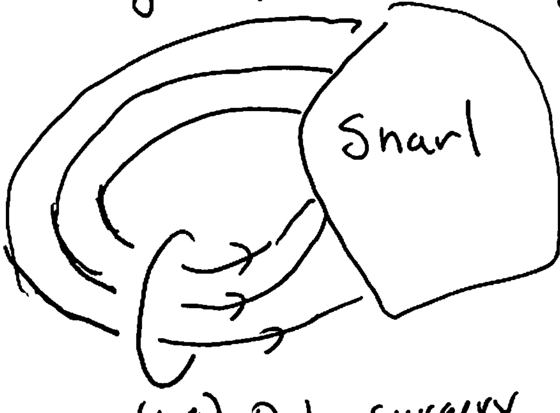
[Show fig 2 of paper]



Max cusp

$\boxed{\text{See pg 4}}$

[Show fig 3] which suggests $|2\sigma(K) - \text{slope}(K)| \le C_2 \, \text{Vol}(K)$



(1,q) Dehn surgery

slope(K(q)) almost const.

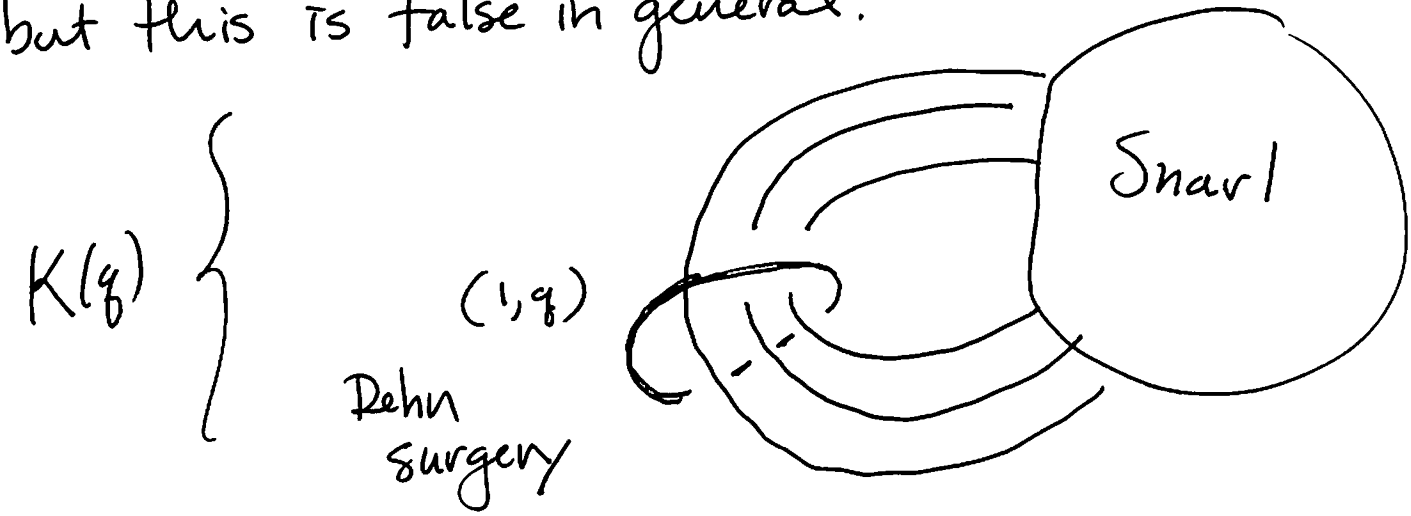
$\sigma(K(q))$ grows like q/q

[Show fig 12 and Conj 7.4]

which suggests

$$|2\sigma(K) - \text{slope}(K)| \leq C_2 \,\text{Vol}(K)$$

but this is false in general:



Has slope(K(q)) almost constant.

$\sigma(K(q))$ grows like $4q$.

[Show fig 12 and Conj 7.4]