

## Math 416: HW 4 due Friday, September 30, 2022.

Webpage: <http://dunfield.info/416>

Office hours: Wednesday 2:30–3:30pm and Thursday 3:00–4:00pm; other times possible by appointment. My office is 378 Altgeld.

Textbooks: In the assignment, the main text is abbreviated as follows:

[FIS] Freidberg, Insel, Spence, *Linear Algebra*, 4th or 5th edition, 2002 or 2019.

### Problems:

- Section 2.1 of [FIS], Problem 1.
- Section 2.1 of [FIS], Problems 2 and 3.
- Section 2.1 of [FIS], Problem 9 (a, b, c).
- Section 2.1 of [FIS], Problem 10.
- Section 2.1 of [FIS], Problems 18.
- Let  $V, W$  be vector spaces, with  $\dim(V) = n$ ,  $\dim(W) = m$ , and  $n > m$ .
  - Show that there is no one-to-one linear transformation  $T: V \rightarrow W$ .
  - Show that there is no onto linear transformation  $T: W \rightarrow V$  (notice that  $V, W$  have flipped in this expression!)
  - Show that a linear map  $T: V \rightarrow W$  need not be onto by giving an example where it is not.Hint: See Appendix B of [FIS] for the definitions of “onto” and “one-to-one” and consult Theorems 2.4 and 2.5 in §2.1 of [FIS].
- We define the linear transformation  $T_\theta: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  to be rotation counter-clockwise about the origin through angle  $\theta$ . Let  $T_x$  be the transformation that reflects in the  $x$ -axis.
  - Write down the matrices of  $T_\theta$  and  $T_x$  with the respect to the standard basis  $\beta = \{e_1, e_2\}$  for  $\mathbb{R}^2$ .
  - Show that for  $\theta \in (0, \pi) \cup (\pi, 2\pi)$  one has
$$T_x \circ T_\theta \neq T_\theta \circ T_x.$$
  - Next, show that there is some angle  $\psi$  such that
$$T_x \circ T_\psi = T_\theta \circ T_x.$$
What is the relationship between  $\theta$  and  $\psi$ ? Discuss the geometric meaning of this computation.
- Section 2.2 of [FIS], Problem 2 (a, b, c).
- Section 2.2 of [FIS], Problem 3.
- Section 2.2 of [FIS], Problem 5.