# Math 418: HW 5 due Wednesday, March 2, 2022.

**In class Midterm Exam:** The in class midterm will be Monday, March 7. The test will cover Chapters 8, 9, and 13 of the text. Problems will be similar to, but easier than, the homework problems with the likely addition of some true/false questions. It will be closed book, but you can bring one sheet of standard-sized paper on which you may write/copy/print anything you think might be helpful to you. You can use both sides, but must be able to read it without special equipment, so no jeweler's loups.

**Webpage:** http://dunfield.info/418

**Office hours:** Monday and Tuesday from 1:30–2:30pm; other times possible by appointment.

1. Let $K_1$ and $K_2$ be subfields of some ambient field $L$ which both contain a subfield $F$. Suppose that each $K_i$ is the splitting field of a polynomial $f_i \in F[x]$. Prove that $K_1 \cap K_2$ is also the splitting field of some polynomial in $F[x]$.

   **Hint:** Problem 5 from HW #4.

2. Find all irreducible polynomials of degree 1, 2, and 4 over $\mathbb{F}_2$. Check that their product is $x^{16} + x$.

3. Suppose $p$ is prime. Prove that $f(x)^p = f(x^p)$ for every polynomial $f(x) \in \mathbb{F}_p[x]$.

4. Suppose $K$ is an extension of a perfect field $F$. Suppose that $f(x) \in F[x]$ has no repeated irreducible factors in $F[x]$. Prove that it also has no repeated irreducible factors in $K[x]$.

5. A field element $\zeta$ is called a root of unity if $\zeta^n = 1$ for some $n > 0$. Using the results of Section 13.6, prove that if $K$ is a finite extension of $\mathbb{Q}$ then it contains only finitely many roots of unity.