Math 416: HW 1 due Friday, January 26, 2024.

Course webpage: http://dunfield.info/416

Office hours: Wednesday 2:30–3:30pm and Thursday 2:00–3:00pm; other times possible by appointment. My office is 378 Altgeld.

Textbooks: In the assignment, the two texts are abbreviated as follows:

[FIS] Freidberg, Insel, Spence, *Linear Algebra*, 4th or 5th edition, 2002 or 2019.

[B] Breezer, *A First Course in Linear Algebra*, Version 3.5, 2015.

Problems:

1. Problem 1 from Section 1.2 of [FIS]. You do not need to justify your answers.

2. Prove the following statements, which are Corollaries 1 and 2 of Section 1.2 of [FIS]. In both cases, V is a vector space over the real numbers.

   (a) The vector 0 required by axiom (VS 3) is unique.

   (b) For each x in V, there is only one y in V satisfying $x + y = 0$.

3. Let V be all pairs (a_1, a_2) where a_1 and a_2 are in $\mathbb{R}$. Define addition of elements of V coordinatewise, and define scalar multiplication by

   $$c(a_1, a_2) = \begin{cases} (0,0) & \text{if } c = 0 \\ \left(\frac{a_1}{c}, \frac{a_2}{c}\right) & \text{if } c \neq 0 \end{cases}$$

   Is V a vector space over $\mathbb{R}$ with these operations? Justify your answer.

4. Problems 8 and 9 from Section 1.3 of [FIS].

5. A square matrix A is called *upper triangular* if all entries lying below the diagonal are 0, that is, $A_{ij} = 0$ whenever $i > j$. Show that the upper triangular matrices form a subspace of $M_{n \times n}(\mathbb{R})$.

6. For a nonempty set S, we use $\mathcal{F}(S, \mathbb{R})$ to denote the set of all functions from S to $\mathbb{R}$; as described in Example 3 on page 9 of [FIS], this is a vector space over $\mathbb{R}$. Fix a point s_0 in S and consider the subset W of $\mathcal{F}(S, \mathbb{R})$ consisting of all functions where $f(s_0) = 0$.

   (a) Show that W is a subspace of $\mathcal{F}(S, \mathbb{R})$.

   (b) Consider instead the subset where $f(s_0) = 1$. Is this also a subspace? Justify your answer.

7. Parts (a), (b), and (c) of Problem 2 of Section 1.4 of [FIS].