Math 416: HW 5 due Friday, March 1, 2024.

Webpage: http://dunfield.info/416

Office hours: Wednesday 2:30–3:30pm and Thursday 2:00–3:00pm; other times possible by appointment. My office is 378 Altgeld.

Textbook: In the assignment, the main text is abbreviated as follows:

[FIS] Freidberg, Insel, Spence, *Linear Algebra*, 4th or 5th edition, 2002 or 2019.

Problems:

1. Section 2.3 of [FIS], Problem 1.
2. Section 2.3 of [FIS], Problem 2.
3. Give matrices $A, B \in M_{2\times 2}(\mathbb{R})$ where AB is the zero matrix but BA is not.
4. Section 2.3 of [FIS], Problem 3.
5. Section 2.3 of [FIS], Problem 4 (a, b).
6. Section 2.4 of [FIS], Problem 1.
7. Suppose A and B are invertible $n \times n$ matrices.
   (a) Prove that $(AB)^{-1} = B^{-1}A^{-1}$.
   (b) Prove that $(A^t)^{-1} = (A^{-1})^t$
8. Let V be a finite-dimensional vector space over $\mathbb{R}$ with basis β, and set $n = \dim V$. Consider the linear transformation $\phi_\beta \colon V \to \mathbb{R}^n$ defined by $\phi_\beta(v) = [v]_\beta$.
   (a) Prove that ϕ_β is an isomorphism.
   (b) Use part (a) to show that any two vector spaces of dimension n are isomorphic. This gives an alternate proof of Theorem 2.19 of [FIS].
9. (a) Let A and B be $n \times n$ matrices such that AB is invertible. Prove that both A and B are invertible.
   (b) Give an example of two noninvertible matrices whose product is invertible. Hint: Look at non-square matrices.
   (c) Prove or give a counterexample: If A and B are nonzero $n \times n$ matrices with AB the zero matrix then A is not invertible.
10. Find the inverse of the following matrix, and give a direct check of your answer.

$$A = \begin{pmatrix} 4 & -2 & -5 \\ -4 & 1 & 4 \\ -3 & 1 & 3 \end{pmatrix}$$