Math 416: HW 6 due Friday, March 8, 2024.

Webpage: http://dunfield.info/416

Office hours: Wednesday 2:30–3:30pm and Thursday 2:00–3:00pm; other times possible by appointment. My office is 378 Altgeld.

Problems:

1. Suppose $T\colon V \to W$ is a linear transformation between finite-dimensional vector spaces, and let $\beta = \{v_1, \ldots, v_n\}$ be a basis for V. Prove that T is an isomorphism if and only if $\gamma = \{w_1, \ldots w_n\}$ where $w_i = T(v_i)$ is a basis for W.

   Hint: We did part of this in class.

2. Section 2.5 of [FIS], Problem 1.

3. Section 2.5 of [FIS], Problem 2 (a–c) and Problem 3 (c) and (d).

4. Section 2.5 of [FIS], Problem 6 (a) and (c).

5. Section 2.5 of [FIS], Problem 7.

6. Compute the determinants of the following matrices:

$$\text{(a)} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad \text{(b)} \begin{pmatrix} -4 & 2 \\ 3 & -4 \end{pmatrix} \qquad \text{(c)} \begin{pmatrix} 2 & 3 \\ -2 & -3 \end{pmatrix}$$

7. Suppose $A \in M_{2\times 2}(\mathbb{R})$.

   (a) Show that $\det(A) = \det(A^t)$.

   (b) Show that if B is obtained from A by swapping the two rows, then $\det(B) = -\det(A)$.

   (c) How does the determinant change if instead you swap the columns of A?

   (d) If B is also in $M_{2\times 2}(\mathbb{R})$, prove that $\det(AB) = \det(A)\det(B)$.

8. Section 4.1 of [FIS], Problem 10.

9. Section 4.2 of [FIS], Problems 5 and 11.

10. Section 4.2 of [FIS], Problem 21.