Math 416: HW 8 due Friday, April 5, 2024.

NOTE: No office hours this week.

1. Let $\mathbb{C}$ denote the field of complex numbers, as discussed in detail in Appendix D of [FIS]. As with any field, we can consider vector spaces, linear transformations, and matrices over $\mathbb{C}$ rather than over our usual field $\mathbb{R}$.

   (a) The complex numbers $\mathbb{C}$ can be viewed as a vector space over either $\mathbb{C}$ or $\mathbb{R}$ with the usual scalar multiplication. Prove that $\mathbb{C}$ has dimension 1 as a vector space over $\mathbb{C}$ but has dimension 2 as a vector space over $\mathbb{R}$. In each case, give an explicit basis.

   (b) Since $\mathbb{R}$ is a subset of $\mathbb{C}$, if V is a vector space over $\mathbb{C}$ then it is also a vector space over $\mathbb{R}$: just use the same scalar multiplication but restricted to scalars in $\mathbb{R}$. If V has dimension n as a vector space over $\mathbb{C}$, prove that it has dimension $2n$ as a vector space over $\mathbb{R}$. Hint: Use Theorem 2.19 from [FIS] to reduce to the case where V is just $\mathbb{C}^n$.

   (c) Diagonalize the following matrices over $\mathbb{C}$ by giving a $Q \in M_{2\times 2}(\mathbb{C})$ so that $Q^{-1}AQ$ is diagonal.

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$$

2. Section 5.1 of [FIS], Problem 1.

3. Section 5.2 of [FIS], Problem 1.

4. Section 5.2 of [FIS], Problem 2, parts (e) and (g).

5. Section 5.2 of [FIS], Problem 3, parts (a) and (d).

6. Prove that similar matrices have the same characteristic polynomial.

7. Section 5.2 of [FIS], Problem 7.

8. If A is a square matrix prove that A and A^t have the same eigenvalues. Do they have the same eigenvectors? Either prove they do, or give a counterexample.

9. Suppose that A in $M_{n\times n}(\mathbb{R})$ has two distinct eigenvalues λ_1 and λ_2, and that $\dim(E_{\lambda_1}) = n-1$. Prove that A is diagonalizable.

10. Section 5.3 of [FIS], Problem 6. This problem differs slightly between the 4th and 5th edition, please do this version:

    In the week beginning June 1, 30% of the patients who arrived by helicopter at a hospital trauma unit were ambulatory and 70% were bedridden. One week after arrival, 60% of the ambulatory patients had been released, 20% remained ambulatory, and 20% had become bedridden. After the same amount of time, 10% of the bedridden patients had been released, 20% had become ambulatory, 50% remained bedridden, and 20% had died. Determine the percentages of helicopter arrivals during the week of June 1 who were in each of the four states one week after arrival. Assuming that the given percentages continue in the future, also determine the percentages of these patients who eventually end up in each of the four states.