Math 20 Final Review Problems - Spring 1999

These problems are collected from past final exams and reviews.

Problems from Fall 1998 Final Exam

F1) Find a vector **v** in span $\left\{\begin{bmatrix}1\\-1\\0\end{bmatrix},\begin{bmatrix}2\\2\\-1\end{bmatrix}\right\}$ which also lies in span $\left\{\begin{bmatrix}0\\-4\\3\end{bmatrix},\begin{bmatrix}1\\1\\2\end{bmatrix}\right\}$.

F2) Find a matrix **A** with the property that $\mathbf{A}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}7\\5\\3\end{bmatrix}$ and $\mathbf{A}\begin{bmatrix}2\\5\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$.

F3) A spider crawls on the surface $z = x + 2y$, staying as close as possible to a fly. If the fly is at the point (2,2,2), where is the spider?

F4) All of the 120 families in a small community enjoy a yearly vacation to one of three theme parks, call them A, B, and C. Of those that go to park A in a given year, 1/6 will go to park B and 1/6 will go to park C the following year with the rest returning to park A. Similarly, 1/4 of those that had gone to park B will choose park A the following year, 1/2 will go to park C, and the rest will give park B another visit. Of those that go to park C, half will go to park B the following year with the rest returning to park C. Assuming the numbers going to each park eventually stabilize, how many families will then be going to parks A, B, and C?

F5) a) Given the matrix **A** below, find a basis for ker(**A**), the kernel of the linear function described by this matrix.

$$\mathbf{A}=\begin{bmatrix}1&-1&-1&1&1\\-1&1&0&-2&2\\1&-1&-2&0&3\\2&-2&-1&3&4\end{bmatrix}$$

b) Find a vector **v** that is orthogonal to im(**A**). (That is, for all vectors **x** for which **Ax** is defined, **v** should be perpendicular to **Ax**.)

F6) a) Consider the surface S described by the equation $z = xy^2 + x^2 - y^3$. Find an equation for the plane tangent to this surface at the point (1,1,1).

b) If one were to travel with unit speed on the surface S in such a way that the distance from the origin did not change, find a possible velocity vector at the point (1,1,1).

F7) This exercise asks you to construct a matrix **A** for the linear function corresponding to projection of vectors onto the plane $2x + y - 2z = 0$. (This is a plane through the origin and is therefore a subspace of $\mathbf{R}^3$.) It will be helpful to note that if you find the vector projection of any vector **x** in the direction of the normal to this plane (call this projection **p**), then the difference vector **x - p** will be the projection of **x** in the plane.

a) Find the vector projections of the standard basis vectors **i**, **j**, and **k** and use this information to construct the matrix **A**.

b) State what the eigenvalues of the matrix **A** will be. You need not specify the eigenvectors. [Hint: You should be able to answer this without further calculation.]

F8) Find the maximum value of the function $f(x, y) = x + y$ in the region bounded by $x = 0$, $x = 1$, $y = 0$, and $y^3 = (x - 1)^2$.

F9) Given the data shown below, find the parabola through the origin with equation of the form $y = ax^2 + bx$ that best fits this data in the sense of least squares.

x	-2	-1	1	2
y	-5	-3	5	8

F10)a) If we were to use the basis for $\mathbf{R}^2$ given by the vectors $\{\mathbf{w}_1, \mathbf{w}_2\} = \left\{\begin{bmatrix}2\\-1\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right\}$, write down linear equations that relate the coordinates (u, v) relative to this basis to the standard coordinates (x, y). [That is, if a vector $\mathbf{x} = x\mathbf{i} + y\mathbf{j}$ in standard coordinates and $\mathbf{x} = u\mathbf{w}_1 + v\mathbf{w}_2$, find equations that relate (u, v) and (x, y).]

b) Consider the function described in standard coordinates by the expression $z = f(x, y) = x^3 - 3xy + y^2$. If we were to describe this function in new coordinates as $z = f(x(u, v), y(u, v))$, calculate the partial derivatives $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at the point where $(u, v) = (3, 2)$.

F11) A country is partitioned into three regions, call them A, B, and C. In collecting yearly demographic information, we find that 75% of the people in region A will remain the next year while 5% go to B and 10% go to C. Of those in region B, 80% will remain while 10% will go to A and 5% to C. Region C is a port city. Not only will 70% of its population of any given year stay while 20% goes to A and 10% goes to B, but there will be an additional 10,000 immigrants arriving each year.

a) Write down a system of equations or an equation involving matrices and vectors that relates the populations in each region in one year to the populations the previous year.

b) Is there any circumstance in which the populations in each region will be the same from one year to the next? If so, find these populations.

Problems from Fall 1997 Final Exam

1) a) At what rate does the function $f(x, y) = 4x^2 - 9y^2$ change if you move at unit speed away from the point (2, 1) in the direction of the vector $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$?

b) If $g(x, y) = 3x^2 + 2y^2$, at what angle do the contours of f and g intersect at the point (2, 1)?

c) What is the equation of the tangent plane to the graph of $z = f(x, y)$ at the point where $x = 2$ and $y = 1$?

2) Given the line $(x, y, z) = (2 + 7t, 1 + 2t, 4 - t)$ in $\mathbf{R}^3$ and the point (-2, 2, 6),

a) find an equation for the plane containing both the line and the point.

b) find the shortest distance from the point (8, 0, -8) to the plane that you just found.

3) a) Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & k & k^2 \\ k & k^2 & k^3 \end{bmatrix}$ where k is a constant. Find the kernel of $\mathbf{A}$, i.e. all solutions of the equation $\mathbf{Ax} = \mathbf{0}$, and give a basis for the kernel. (You may have to look at cases for a complete answer.)

b) Are there any choices of the constant k such that the vector $\mathbf{x} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$ is a solution of $\mathbf{Ax} = \mathbf{0}$?

4) A matrix can described geometrically by how it acts on any basis of vectors. Given the basis for $\mathbf{R}^3$ consisting of the vectors $\{\mathbf{v}_1 , \mathbf{v}_2 , \mathbf{v}_3\}$ where $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, and that $\mathbf{Av}_1 = \mathbf{v}_1 - 2\mathbf{v}_2 + \mathbf{v}_3$, $\mathbf{Av}_2 = -2\mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3$, and $\mathbf{Av}_3 = \mathbf{v}_1 + \mathbf{v}_2 - 3\mathbf{v}_3$, determine the matrix $\mathbf{A}$ (relative to the standard basis).

5) The quadratic equation $2x^2 + 2xy + 4y^2 = 98$ represents an ellipse.

a) Find two vectors corresponding to the long and short axes of this ellipse (magnitude of these vectors is not important). Identify which vector corresponds to the long axis and which vector corresponds to the short axis.

b) Find the lengths a and b of the long and short semi-axes of this ellipse. Express your answer to 4 decimal place accuracy.

6) Find all critical points of the function

$$f(x, y) = 8x^3 + 6xy - 3y^2 - 24x - 6y + 5$$

Classify each critical point as either a local minimum, a local maximum, or a saddle point.

7) Find the maximum and minimum values of the function $f(x, y) = 2x + y$ on the ellipse $2x^2 + 2xy + 4y^2 = 98$.

8) In 1940 a county land-use survey showed that 10 % of the county land was urban, 50% was unused and 40% was agricultural. Five years later a follow-up survey revealed that 70% of the urban land had remained urban, 10% had become unused, and 20% had become agricultural. Likewise, 20% of the unused land had become urban, 60% had remained unused, and 20% had become agricultural. Finally, the 1945 survey showed that 20% of the agricultural land had become unused while 80% remained agricultural. Assuming that the trends indicated by the 1945 survey continue, compute the percentage of urban, unused, and agricultural land in the county in 1950 and the corresponding eventual percentages.

Problems from Spring 1997 Final Exam:

9) Two planes are given by the equations $2x + y + z = 2$ and $x - y - 3z = 4$. Find the point on their line of intersection that is closest to the origin.

10) Consider the 4×5 matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & -2 & 0 & -1 \\ 0 & 3 & 3 & 0 & 3 \\ -2 & -2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 1 & 4 \end{bmatrix}.$$

a) Find a basis for the kernel of **A**. What is its dimension?

b) Find a basis for the image of **A**. What is its dimension? (*The image of* **A** *consists of all vectors of the form* **Av** *where* **v** *is in the domain. This subspace is spanned by the column vectors of the matrix* **A**.)

c) Find all solutions of the linear system **Ax** = **b**,

where **b** is the vector $\mathbf{b} = \begin{bmatrix} 4 \\ -3 \\ -1 \\ 4 \end{bmatrix}$.

11) Find the maximum and minimum values of the function $f(x, y) = 11x + 12y$ on the ellipse $6x^2 + 8xy + 12y^2 = 56$.

12) Consider the <u>surface</u> $z = 6x^2 + 8xy + 12y^2$.

a) Find an equation for the tangent plane to this surface at the point where $x = 3$ and $y = -1$.

b) If you were to travel <u>on this surface</u> in such a way that the distance from the z-axis remains constant, give parametric equations for the line tangent to your path as you pass through the point on the surface where $x = 3$ and $y = -1$.

13) An ellipse is given by the equation:

$$6x^2 + 8xy + 12y^2 = 56.$$

a) Determine vectors whose directions give the long and short (principal) axes of this ellipse.

b) If the area of an ellipse is given by πab where a and b are the lengths of its long and short semi-axes, find the area enclosed by this ellipse.

c) Find the angle between the long axis of the ellipse and the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

14) Let $\mathbf{A} = \begin{bmatrix} -4 & 26 & -6 \\ 0 & 0 & 0 \\ 3 & -21 & 5 \end{bmatrix}$.

a) Find the characteristic polynomial $f_{\mathbf{A}}(\lambda)$ of **A**.

b) Find the eigenvalues and eigenvectors of **A**.

c) Using the results of b) (and without using a calculator to compute $\mathbf{A}^{60}$), find $\mathbf{A}^{60}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

15) There is a relatively unknown world of vampires and werewolves among us. What you may not know is that werewolves can be transformed into vampires and vampires into werewolves. In fact, each year 30% of vampires become werewolves and 10% of werewolves become vampires. Vampires cannot be killed, but werewolves <u>can</u> die and, in fact, lose 10% of their number each year in this way. The only way into this unknown world is to be bitten by a vampire, thus becoming a vampire. Only one in five vampires manage to successfully create a new vampire in a given year. Describe how the population of vampires and werewolves will evolve over time. In particular, determine whether their numbers will stabilize, decrease, or increase and at what rate, and what percentages of this unknown world will be comprised by vampires and by werewolves.

Problems from Fall 1996 Final Exam

16) There are three interconnected rooms in a sleazy apartment. There are many flies buzzing about. During a long weekend, we find that after each hour 1/5 of the flies in the living room will have wandered into the kitchen and 1/5 into the bedroom. Of the flies that were in the bedroom, 1/3 have migrated into the kitchen and 1/6 into the living room. Of the flies in the kitchen the previous hour, 1/3 are now in the bedroom and 1/4 are in the living room. Assuming that the flies continue to exhibit this same behavior each hour (and that the population of flies doesn't change), how many total flies are there in the apartment if the number of flies in the living room is found to be holding steady at 130 flies? How many flies are in the kitchen and in the bedroom?

17) Given the vectors $\mathbf{u} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ and a matrix **A** such that

Au = -**u** + **v** + 2**w**
Av = 2**u** + **v** - **w**
Aw = 3**u** - 2**v** + **w**

What is the matrix **A**?

18) Find all solutions of the following system of linear equations: $\begin{Bmatrix} 3x - 4y - 3z + 2w = 14 \\ -x + y + 2z + 3w = 12 \\ x - 5y - 3z + 2w = 6 \\ -2x + z + w = -2 \end{Bmatrix}$

19) a) Determine whether the three vectors $\mathbf{x}_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} -2 \\ -13 \\ 13 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 4 \\ 2 \\ -5 \end{bmatrix}$ are linearly independent.

b) If $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ are linearly independent, find the volume of the parallelepiped with these three vectors as edges. If $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ are linearly dependent, find an equation for the plane through the point (3, 5, -2) that is parallel to the span of these three vectors.

20) Consider the function $f(x, y) = 3x^2 + 4xy + 5y^2$.

a) Show that the origin is a relative minimum for this function.

b) The contour $f(x, y) = 12$ is an ellipse. The area of an ellipse with semi-axes a and b is given by $A = \pi ab$. Find the area bounded by this ellipse and the angle between the long axis and the x-axis.

c) On the above ellipse, find two points where the tangent line to this ellipse is parallel to the line $3x + 5y = 7$.

21) Consider the two surfaces S_1 and S_2 given, respectively, by the two equations:

$$5x^2 + 4y^2 + 10xy - 2xz - 2yz + 1 = 0$$
$$x^2 + 2y^2 + 4z^2 = 7$$

These two surfaces intersect in a curve that passes through the point (1, -1, -1). Find the directional derivative of the function $f(x, y, z) = 3x + y^2 - 10z$ at this point, in the direction determined by this intersection curve. (Note: Since the curve extends in two directions from this point, only the magnitude of this derivative should be given.)

22) We are convinced that the four (x, y) data points (-3, 1), (0, 2), (2, 4), and (4 ,15) can be well approximated by an exponential function of the form $y = \mathrm{A}e^{kx}$. In order to avail ourselves of least square methods, we take the natural logarithm to get the relation $\ln y = \ln \mathrm{A} + kx$, or $z = a + kx$ if we call $z = \ln y$ and $a = \ln \mathrm{A}$. Find, to three decimal place accuracy, the constants A and k for the exponential function that best fits this data in the sense of least squares.

Additional Review Problems

23) Find all solutions of the linear systems:

a) $$\begin{aligned} x - y + z + w &= 10 \\ 2x + 3y - z &= 4 \\ 4x + 2y - 2w &= 9 \\ -x + 3w &= 5 \end{aligned}$$

b) $$\begin{aligned} -3x + 5y + 2z &= 1 \\ 2x - 2y + z &= 4 \\ x + 2y + z &= 3 \end{aligned}$$

24) Given the line $(x, y, z) = (5 - 2t, 2 + t, -3 + 2t)$ in $\mathbf{R}^3$ and the point (9, 5, -2),

a) find an equation for the plane containing both the line and the point.

b) find the shortest distance from the point (6, 1, 0) to the plane you just found.

25) Given that the matrix **A** has the property that

$$\mathbf{A}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}-7\\-4\end{bmatrix} \text{ and } \mathbf{A}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix},$$

a) find the matrix **A**,

b) find the eigenvalues of **A**,

c) find the corresponding eigenvectors of **A**,

d) find $\mathbf{A}^{-1}$.

26) Here are 3 vectors in $\mathbf{R}^2$:

$\mathbf{u} = (3,1)$, $\mathbf{v} = (4 , -2)$, and $\mathbf{w} = (-3 , -3)$

a) Find the coords. of **w** relative to the basis $S = \{\mathbf{u}, \mathbf{v}\}$.

b) Find the coords. of **u** relative to the basis $T = \{\mathbf{v}, \mathbf{w}\}$.

c) Find a matrix **A** that takes the vectors of the basis S respectively to the vectors of the basis T.

d) If the vector **x** has coordinates relative to the basis T of (10, 5), what are the coordinates of **x** relative to the basis S?

27) Determine whether the 3 vectors $\begin{bmatrix}2\\1\\7\\-5\end{bmatrix}$, $\begin{bmatrix}-3\\-3\\1\\2\end{bmatrix}$, and $\begin{bmatrix}1\\0\\-1\\6\end{bmatrix}$ in $\mathbf{R}^4$ are linearly independent.

28) Consider the matrix $\mathbf{A} = \begin{pmatrix}-15 & 8\\ -24 & 13\end{pmatrix}$:

a) Find all the eigenvalues and eigenvectors of **A**.

b) Compute $\mathbf{A}^7$.

c) Estimate the size of the lower left entry of $\mathbf{A}^{1995}$.

d) Solve the system of differential equations:

$$\frac{d\mathbf{v}}{dt} = \mathbf{A}\mathbf{v}\,; \qquad \mathbf{v}(0) = \begin{pmatrix}2\\-1\end{pmatrix}$$

29) A linear function from $\mathbf{R}^3$ to itself, given relative to the standard basis by a matrix **A**, is described geometrically as follows:

i) This function leaves the line in the direction of the vector $\mathbf{u} = (-1,4,2)$ unchanged (invariant), except that any vector in the direction of this line is doubled in length and reflected into the reverse direction.

ii) The two-dimensional subspace (plane) containing the vectors $\mathbf{v} = (1,1,1)$ and $\mathbf{w} = (3,0,1)$ is preserved by this function, but $\mathbf{A}$ scrambles things up a bit by taking $\mathbf{v}$ to the vector $3\mathbf{v} - \mathbf{w}$ and taking $\mathbf{w}$ to the vector $\mathbf{v} + 2\mathbf{w}$.

a) What is the matrix **A**?

b) If we were to express this same linear function in terms of the basis $B = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ by a matrix $\tilde{\mathbf{A}}$, what would the matrix $\tilde{\mathbf{A}}$ be?

30) A 3 by 3 matrix **A** is defined geometrically by the fact that it fixes the vector $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ and rotates the subspace perpendicular to this vector by 180 degrees.
(Draw a picture! Think about what directions are fixed or reversed.)

a) What are the eigenvalues of the matrix **A**?

b) Find, if possible, a basis of $\mathbf{R}^3$ consisting of eigenvectors of **A** .

c) Find the matrix **A** .

31) One young conoisseur has $900 to spend to build a small wine cellar. She enjoys two vintages in particular: an expensive 1971 French Bordeaux at $30 per bottle and a less expensive 1973 California varietal wine priced at $6. How much of each wine should she purchase if her utility is characterized (strange as this may seem) by the following function?:

$$U(W_F, W_C) = W_F^{\,2/3} W_C^{\,1/3}$$

32) Let $f(x, y) = x^2 y - 4xy + \frac{1}{3} y^3$. Find all critical points of f and identify whether they are relative maxima, relative minima, or saddle points.

33) Suppose that production at a lollipop factory is modelled by the Cobb-Douglas function

$$p(x, y) = 500 x^{0.4} y^{0.8}$$

where x is the number of units of labor and y is the number of units of capital. If the labor costs $60 per unit and the cost of capital is $100 per unit, find the least costly combination of inputs to produce 10,000 units of lollipops and find the minimum cost.

34) Design an open top cylindrical container which holds 50cc and has the minimum possible surface area. What should its dimensions be?

35) a) Find an equation for the plane tangent to the surface $x^2y + 3y^2z + yz^2 = -8$ at the point (3,-2,1).

b) What is the minimum distance from the origin to this tangent plane?

36) What is the directional derivative of the function $f(x, y, z) = x^3 + y^2 + z$ in the direction perpendicular to the surface $xyz + x^2y + 5x^2z = 5$ at the point (-1,2,1)?

37) Two surfaces S_1 and S_2 are described by the equations:

$$S_1: \; xy - x^2 + z^2 = 1$$
$$S_2: \; 3y = 2x^2 + y^3$$

These surfaces intersect in a curve C that contains the point (1,1,1).

a) Find equations of the tangent planes to S_1 and S_2 respectively at the point (1,1,1).

b) Find parametric equations for the line tangent to C at (1,1,1).

c) On surface S_1 , its equation implicitly defines the variable x as a function of the other two variables. Give expressions for the two partial derivatives of this function.

38) Find the quadratic function $f(x) = a + bx + cx^2$ that best fits the points (-1,-2), (-1,-1), (0,0), (1,-1), (1,-2) in the sense of least squares.

39) A country is divided into three regions. Each year, 10% of the residents of region 1 move to region 2 and 5% move to region 3; 15% of the residents of region 2 move to region 1 and 5% to region 3; and 10% of the residents of region 3 move to region 1 and 10% move to region 2. The total population of the country remains constant.

a) Find a matrix that represents the population transitions between regions from year to year.

b) Will the distribution of the population among the three regions tend toward fixed proportions? If so what shall these proportions be?

40) a) Find the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{bmatrix} -7 & 1 & 7 \\ -9 & 1 & 9 \\ -5 & 1 & 5 \end{bmatrix}.$$

b) Calculate $\mathbf{A}^7$. You can verify with a calculator or by brute force, but solution should use the above results.

41) A certain fish population has two age classes, and a Leslie matrix $\mathbf{L} = \begin{pmatrix} 1 & 3/2 \\ 1/2 & 0 \end{pmatrix}$.

a) If the population vector at the start of the n-th growth period is $\mathbf{p}_n$, and if $\mathbf{p}_{n+1} = \mathbf{L}\mathbf{p}_n$, what happens to $\mathbf{p}_n$ as $n \to \infty$.

b) Now suppose that, in addition, at the end of each growth period, a fraction h_1 of the first age class is

caught and killed, and h_2 of the second.
Show that $\mathbf{p}_{n+1} = (\mathbf{I}-\mathbf{H})\mathbf{L}\mathbf{p}_n$ where **I** is the identity matrix and $\mathbf{H} = \begin{pmatrix} h_1 & 0 \\ 0 & h_2 \end{pmatrix}$.

c) Now suppose that $h_1 = h_2 = h$. Is there any value of h which will allow the fish population to come to a stable state? If not, explain why not; if so, find the value of h and the stable state.

42) A video rental company has two locations, one in Boston and one in Cambridge, and borrowed videos can be returned to either location at no extra cost. There is an 80% probability that a video that is in Boston on Sunday will be in the Boston store the following Sunday, and a 70% probability that a video that is in Cambridge on Sunday will still be in Cambridge the following Sunday. Suppose the company stocks 10,000 videos which are initially all in Boston.

a) How will the videos be distributed between the two locations eventually? Set up the appropriate matrix equation and explain your reasoning.

b) How many videos do you expect at each location after 5 weeks?

43) Let's say that an economic utility function U is a function of four economic variables x, y, z, and w (all ≥ 0) and is given by $U(x, y, z, w) = 2x^2 + 3y^2 + 4z^2 + xw$. Let us further suppose that, due to limits on budget and resources, we are forced to live with the fact that $x + y + 2z + 3w \leq 25$ and $2x + y + 2z \leq 20$. What choices of x, y, z, and w will yield maximum utility?

44) Given the system of differential equations:

$$\begin{Bmatrix} \dfrac{dx}{dt} = 5x - 3y \\ \dfrac{dy}{dt} = 9x - 7y \end{Bmatrix}$$

a) Determine the general solution for $x(t)$ and $y(t)$.

b) Find the particular solution for the case where $x(0) = 1$ and $y(0) = 0$.

c) Solve the same system with initial conditions $x(0) = -1$ and $y(0) = 2$.

d) Give a rough sketch of the solutions (for $t \geq 0$) from parts a and b.