

Mathematical Truths: Experiment, Proof, and Understanding

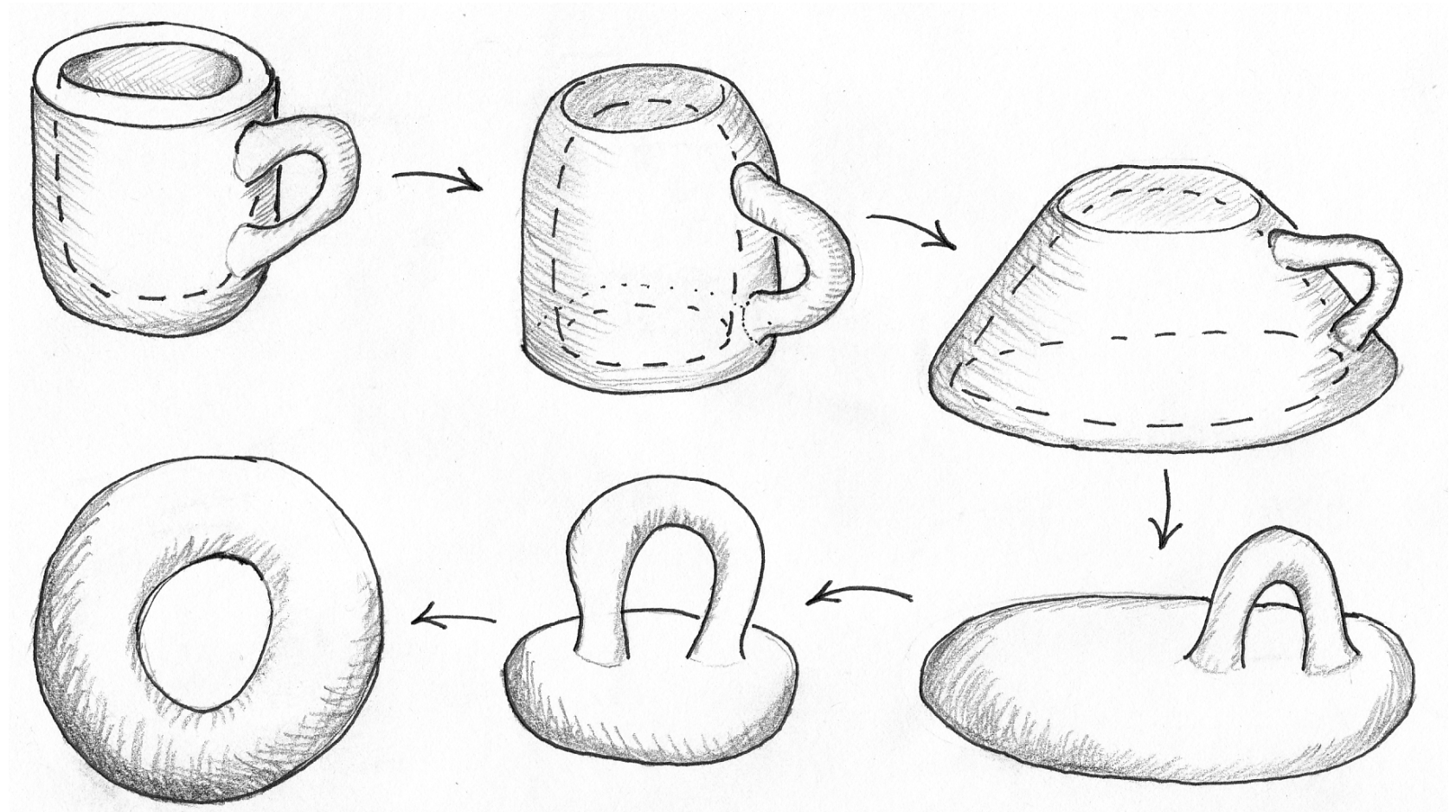
Nathan M. Dunfield

University of Illinois

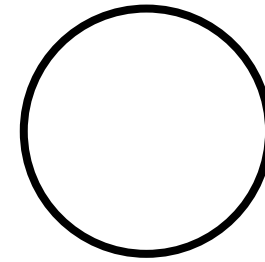
What is topology? The study of objects up to rubbery stretching.

What is a topologist?

Someone who can't tell a coffee cup from a doughnut.

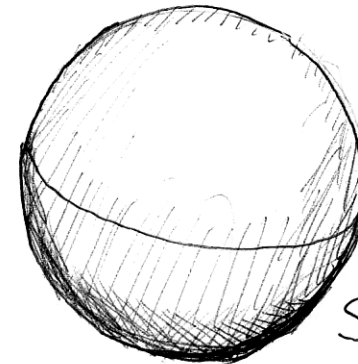
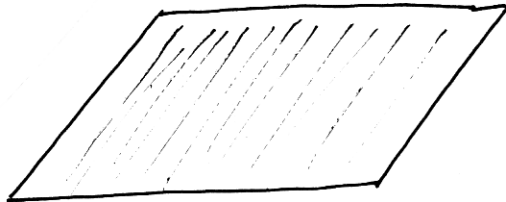


1-manifold: Locally like a line.



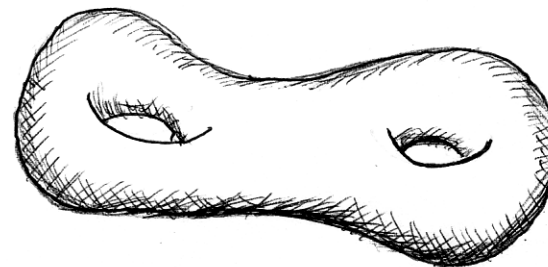
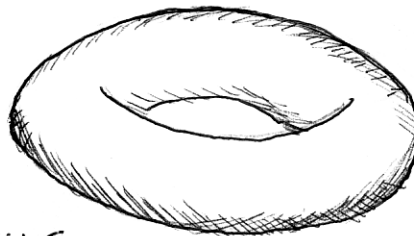
2-manifold (surface): Locally like a plane.

plane



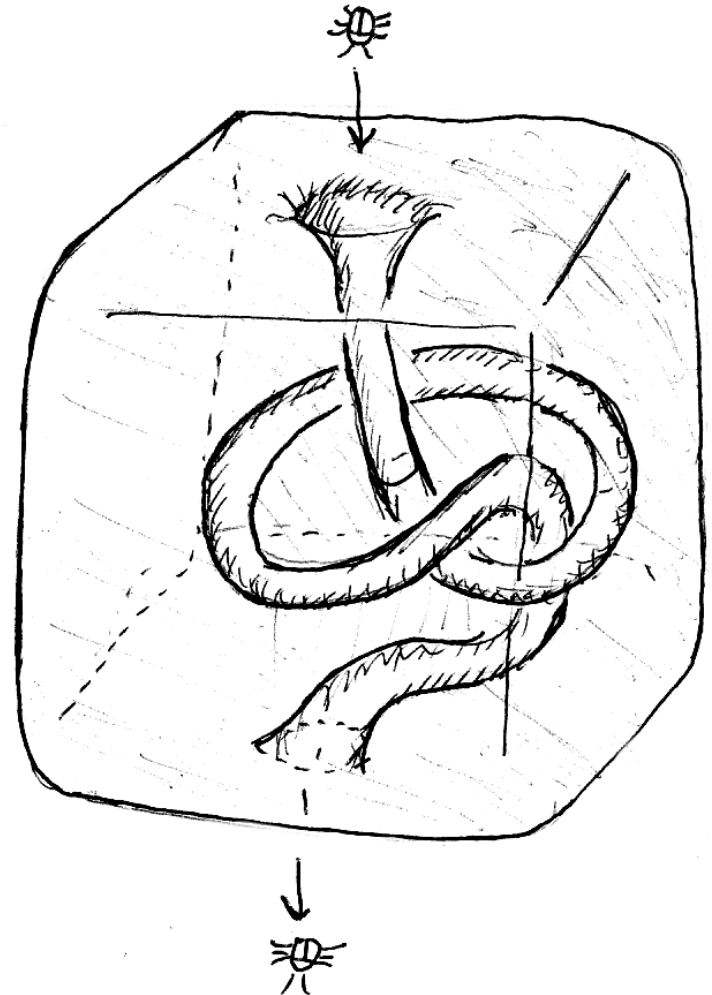
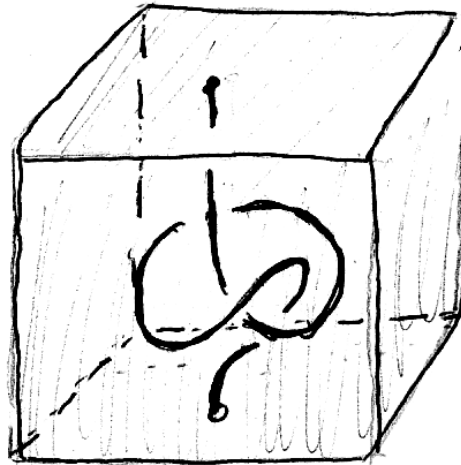
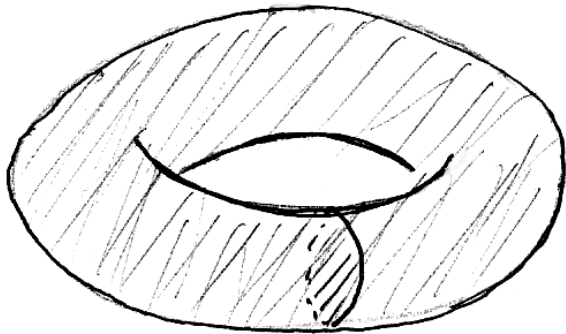
Sphere

torus

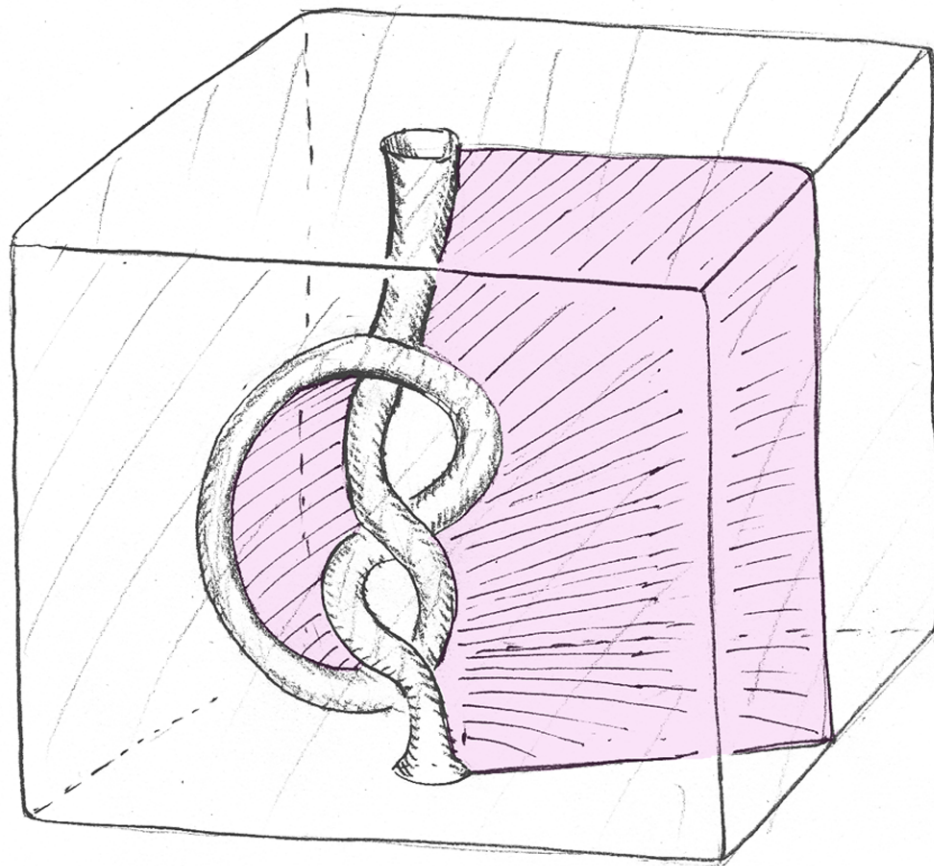
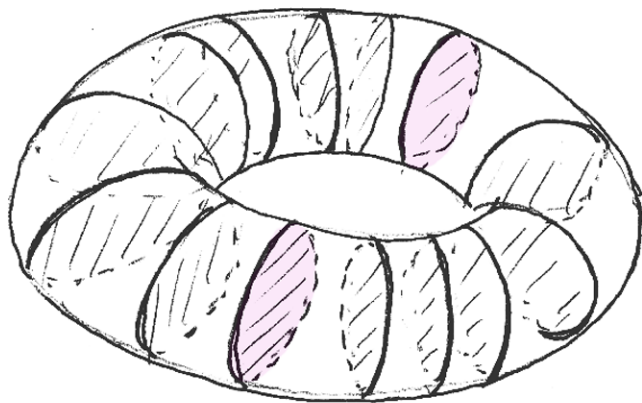


3-manifold: Locally like 3-dimensional space.

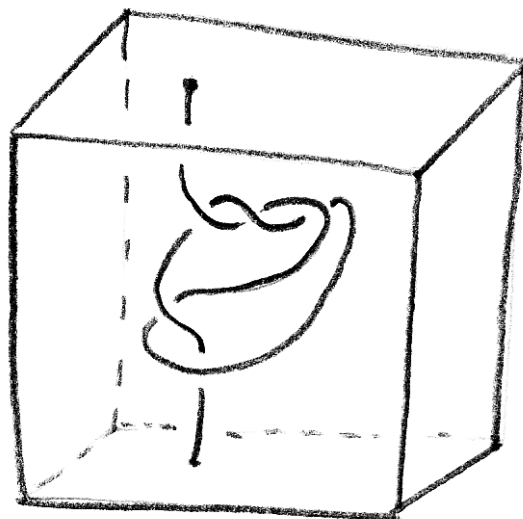
3-manifolds with boundary a torus:



A 3-manifold is *fibered* if it is swept out by surfaces. For instance, the doughnut is a circle's worth of disks.

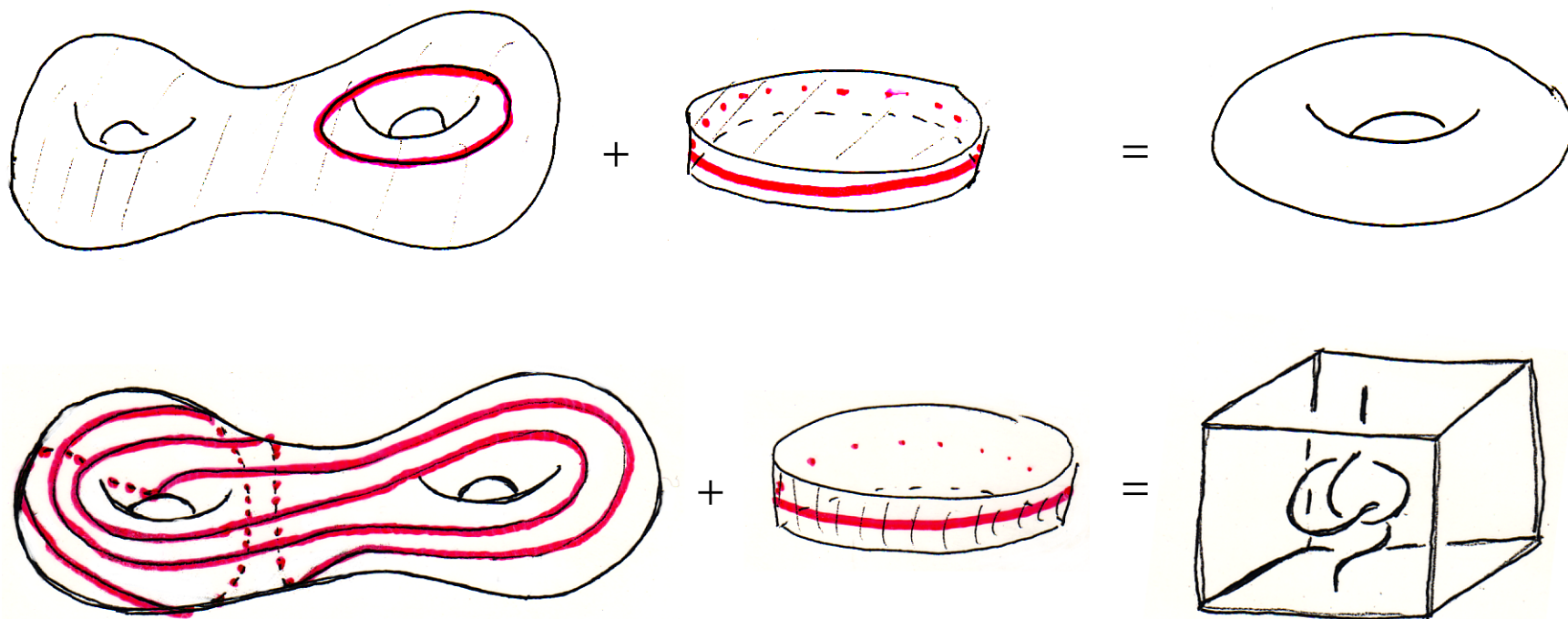


Not every 3-manifold fibers. For instance, this one does not:



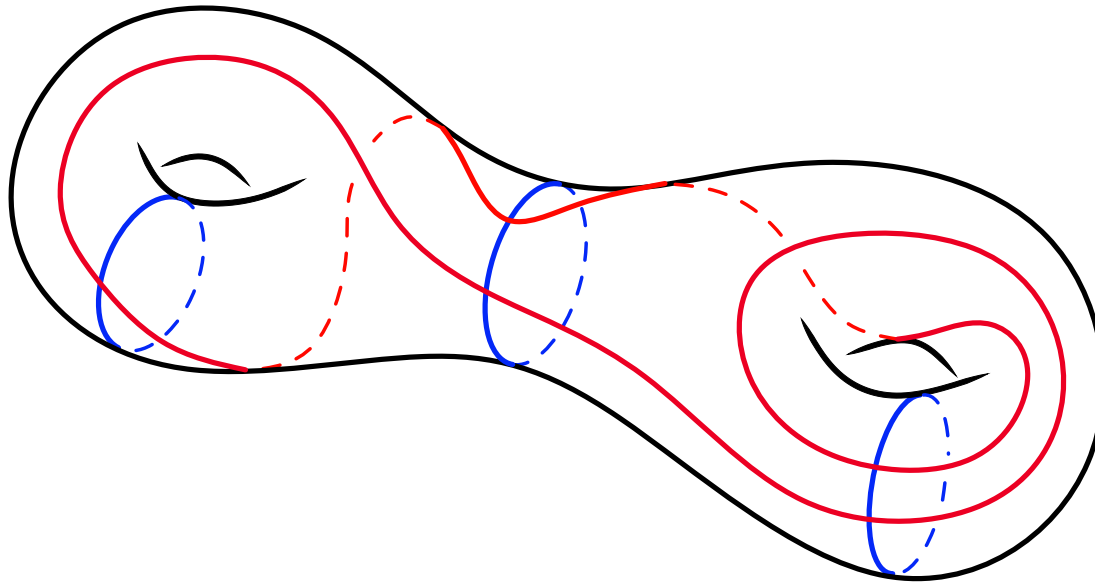
Main Question: *How common is it for a 3-manifold to fiber?*

A special kind of 3-manifold:



Revised question: How common is it for such a 3-manifold to fiber?

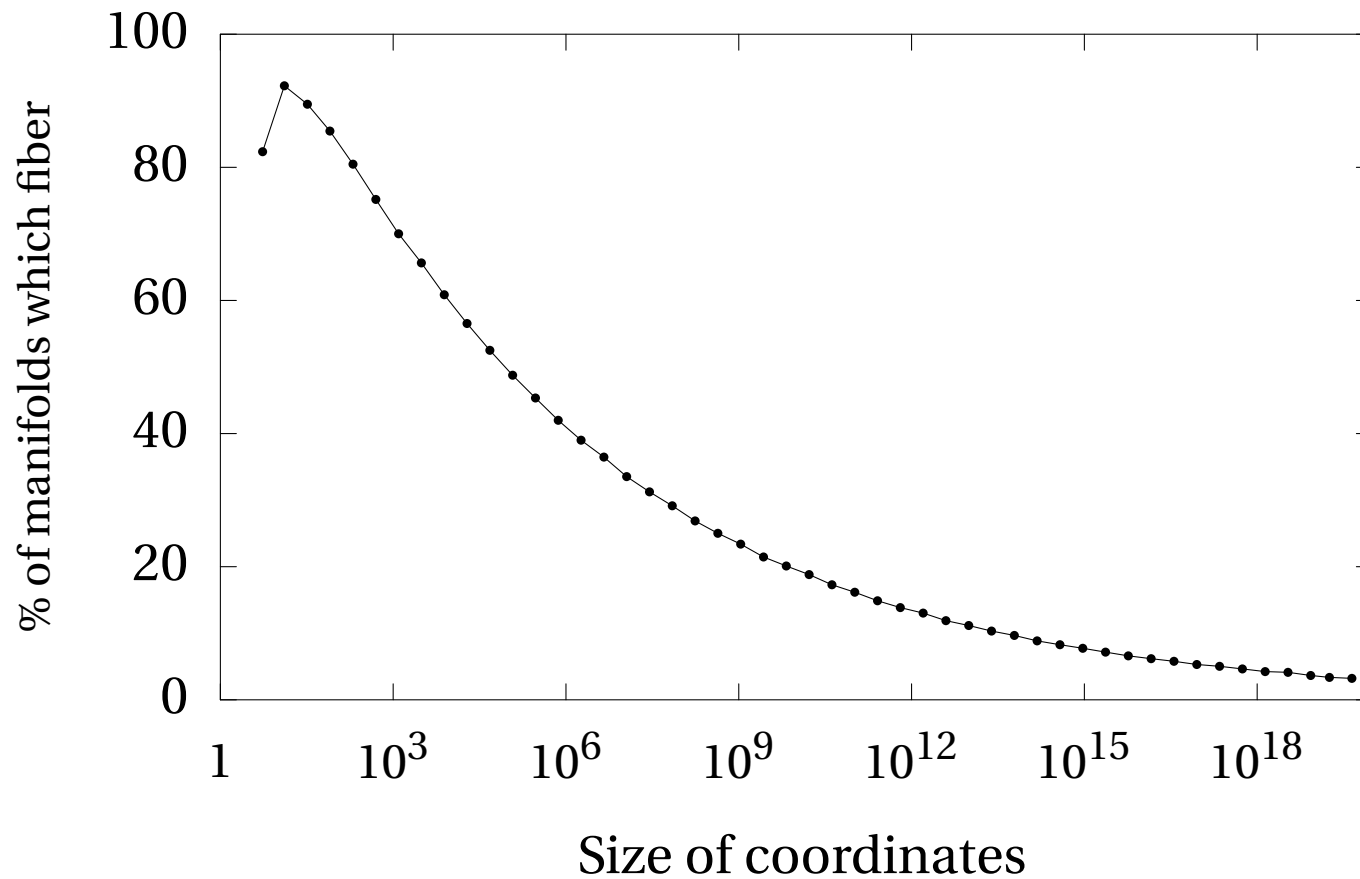
Coordinates for curves on a surface:



Weights:	1	2	2
Twists:	0	1	-1

Procedure which computes whether the associated manifold fibers:
Stallings (1962) + K. Brown (1987).

Experimental Results:



Q1: *Does a typical 3-manifold fiber?*

A: No, at least for the type of 3-manifold we've looked at. In particular, the more complicated the manifold, the closer the odds of it fibering is to 0%.

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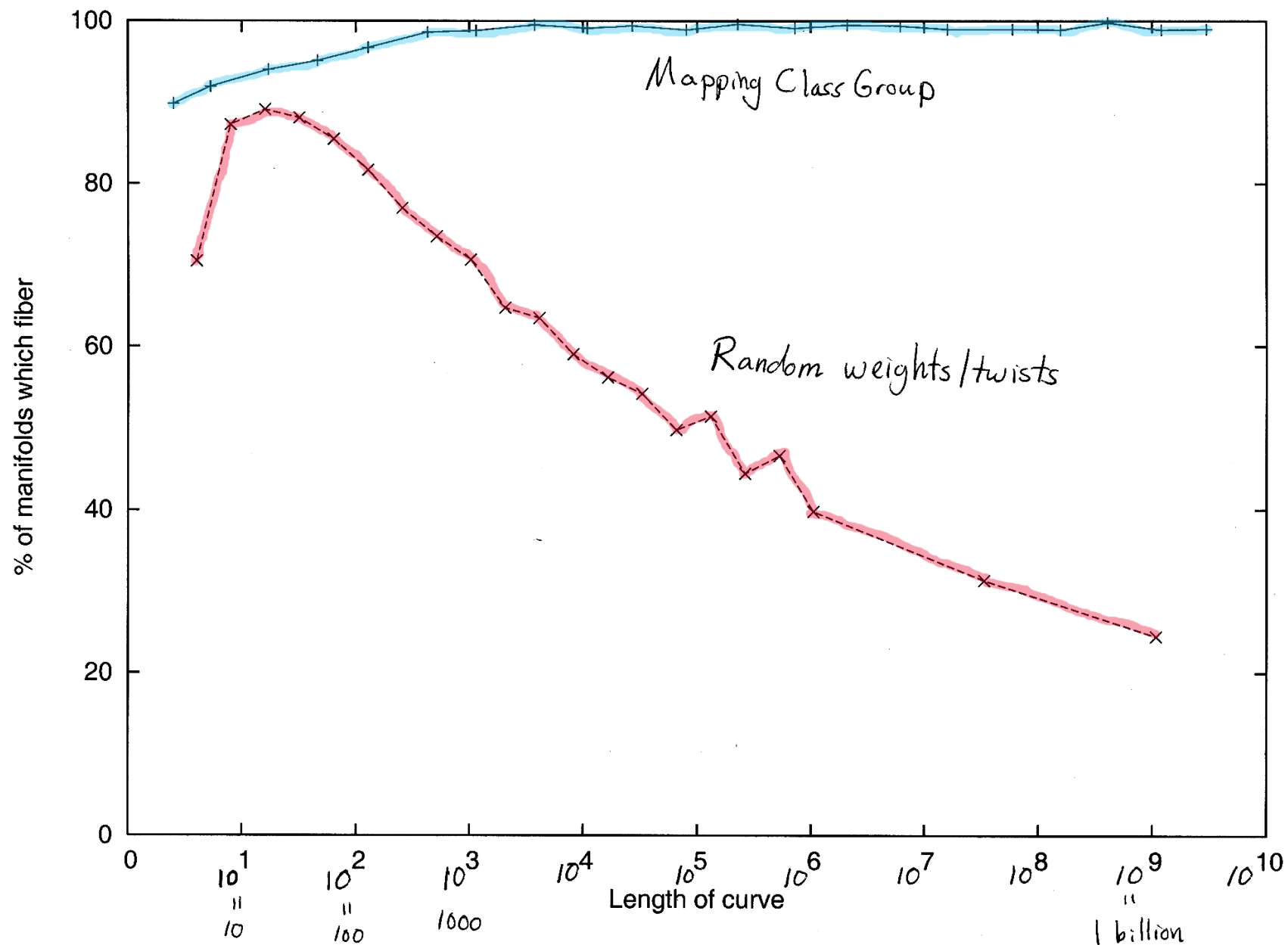
Q2: *Why?*

Q1: *Does a typical 3-manifold fiber?*

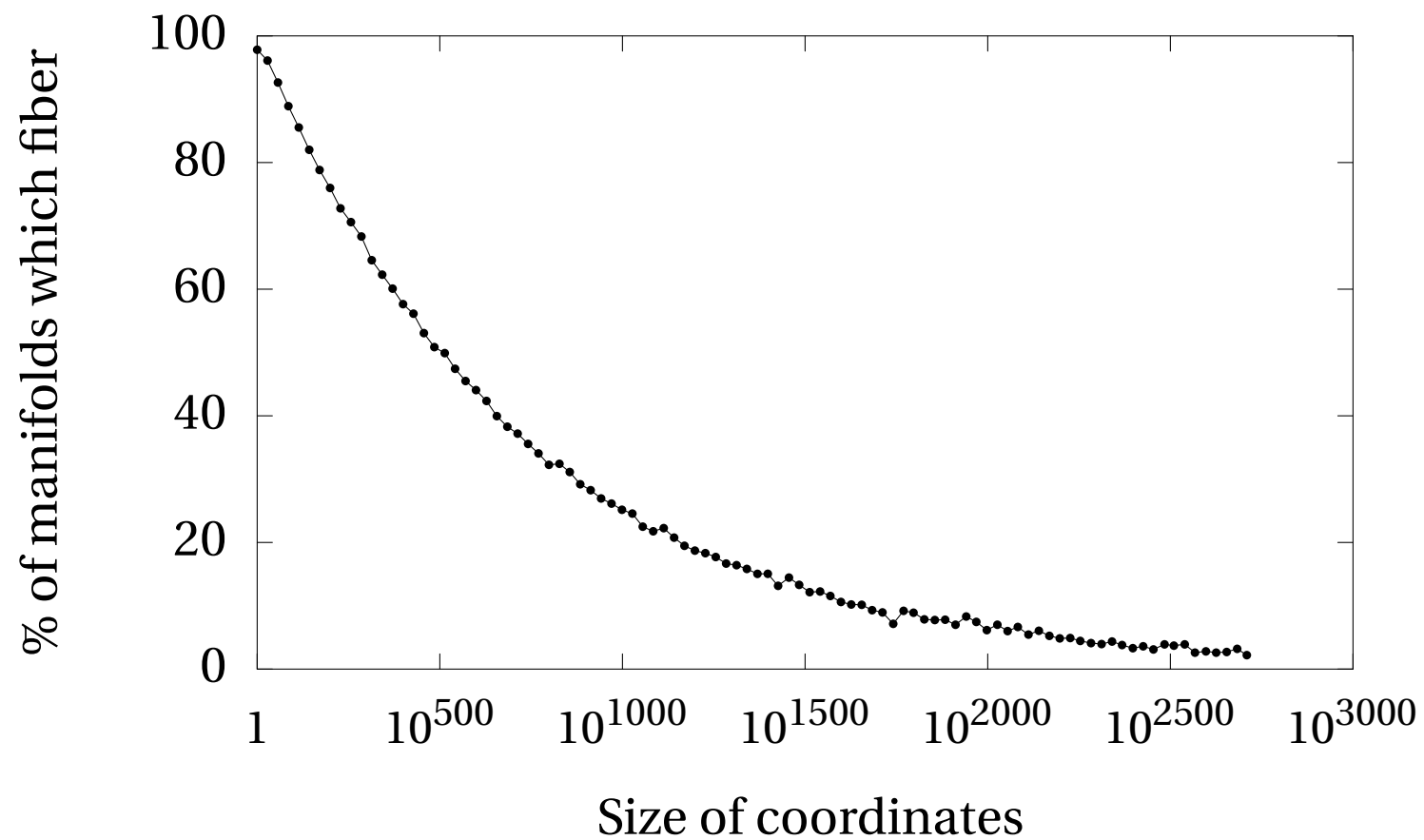
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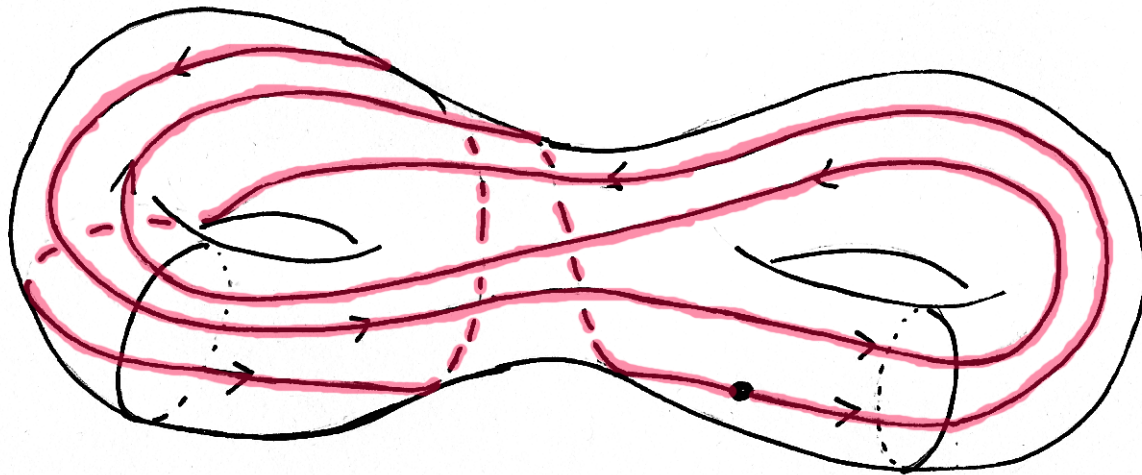
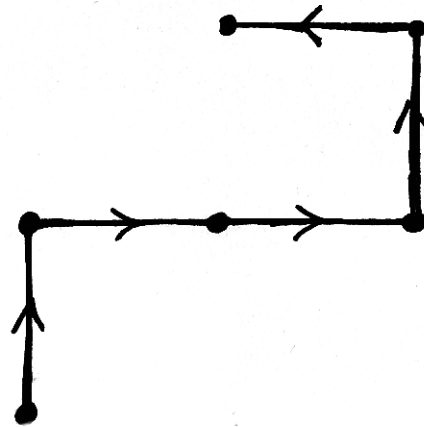
Q2: *Why?*

Q3: *How can we prove this?*



Alternate sampling method

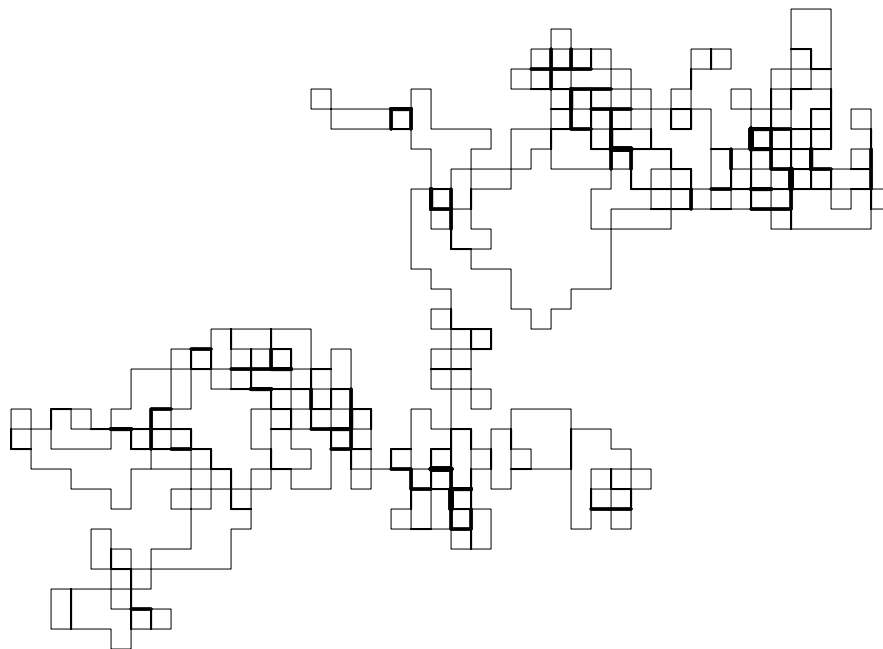
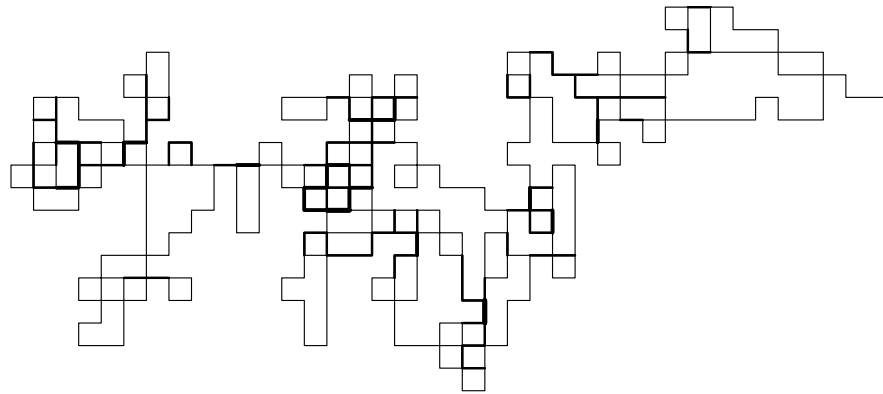




left | right

down | up

Two random walks in the plane





Good books about topology:

- Colin Adams, *Why Knot? An Introduction to the Mathematical Theory of Knots*, 2004. ISBN 1-931914-22-2
- Jeffrey Weeks, *The Shape of Space*, 2001. ISBN 0-8247-0709-5

Original Sources:

- N. M. Dunfield and D. P. Thurston, *A random tunnel number one 3-manifold does not fiber over the circle*, *Geometry and Topology* (2006) 2431–2499. <http://arxiv.org/abs/math/0510129>
- This presentation: <http://dunfield.info/preprints/>