Warwick School: Nathan PS 4   Friday, September 15, 2017.

For further references and links: http://dunfield.info/warwick2017

1. Suppose $\mathcal{T}$ is an ideal triangulation of a 3-manifold M and $z \in \mathbb{C}^n$ satisfies Thurston's theorem and so gives a complete hyperbolic structure on M. How would you use this information to find the holonomy representation $\rho \colon \pi_1(M) \to \mathrm{PSL}_2\mathbb{C}$ of this hyperbolic structure? In particular, you should have

$$M = {}_{\Gamma}\backslash^{\mathbb{H}^3} \quad \text{where } \Gamma = \rho\big(\pi_1(M)\big).$$

   Hint: To get a presentation of $\pi_1(M)$, look at the 2-skeleton of the dual cellulation to $\mathcal{T}$.

2. Prove the theorem on page 4 of the notes for Lecture 4 for any C^1 function $f \colon \mathbb{R} \to \mathbb{R}$. Hint: The Mean Value Theorem should be your friend.

3. SageMath and SnapPy are friends.

   (a) Find the first nontrivial knot in `HTLinkExteriors` which has the same Alexander polynomial as the unknot.

   (b) Let M be the knot exterior you found in part (a). Via the verify module in SnapPy, use interval arithmetic to rigorously prove that this manifold is indeed hyperbolic. Consequently, the corresponding knot is not the unknot.

   (c) Find exact expressions for the tetrahedra shapes of M, which live in some number field. Hint: See http://snappy.computop.org/snap.html