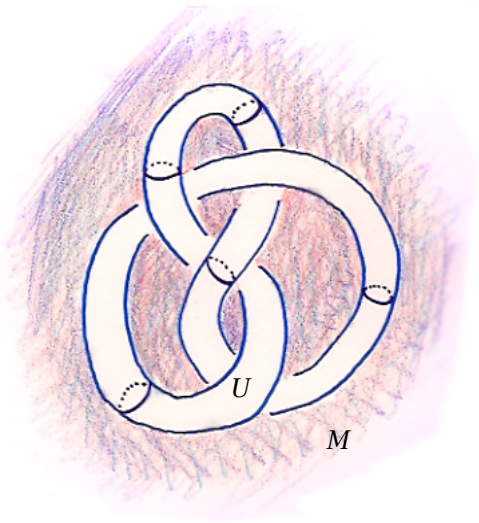


Math 526: HW 4 due Wednesday, October 22, 2014.

1. Hatcher §3.3: Do either #21 or #22, your choice.
2. A knot  $K$  in  $S^3$  is the image of a smooth embedding of  $S^1 \hookrightarrow S^3$ . Prove there always exists an embedded orientable surface with boundary  $\Sigma$  in  $S^3$  where  $\partial\Sigma = K$ . Such a surface is called a *Seifert surface* for  $K$ .

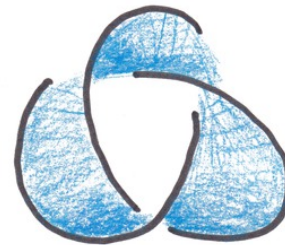
Hint: Choose a neighborhood  $N$  of  $K$  which is homeomorphic to  $S^1 \times D^2$ , let  $U$  be its interior, and take  $M = S^3 \setminus U$ , which is a compact 3-manifold with boundary a torus. Apply Alexander and Poincaré-Lefschetz duality to understand the homology and cohomology of  $M$ .



Examples:



Non-examples:



Same knot, turns out.

3. Hatcher §4.1: #2.
4. Hatcher §4.1: #4.
5. Hatcher §4.1: #11.
6. Hatcher §4.1: #15.