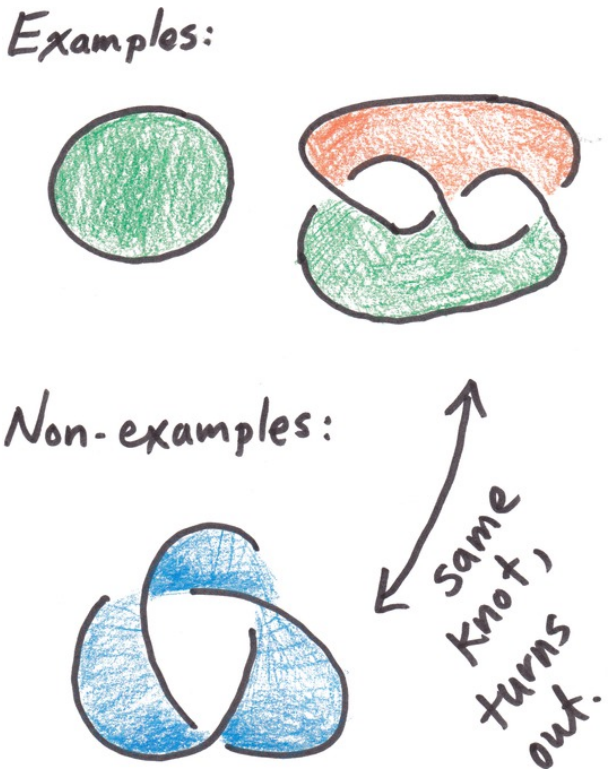
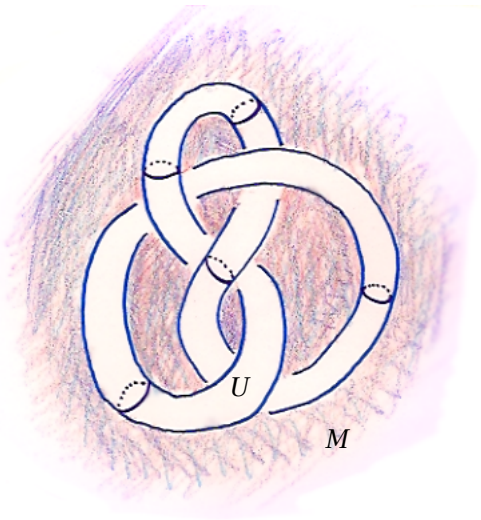
Math 526: HW 4 due Wednesday, October 20, 2021.

1. Hatcher §3.3: Do either #21 or #22, your choice.

2. A *knot* K in S^3 is the image of a smooth embedding of $S^1 \hookrightarrow S^3$. Prove there always exists an embedded orientable surface with boundary Σ in S^3 where $\partial\Sigma = K$. Such a surface is called a *Seifert surface* for K.

   Hint: Choose a neighborhood N of K which is homeomorphic to $S^1 \times D^2$, let U be its interior, and take $M = S^3 \setminus U$, which is a compact 3-manifold with boundary a torus. Apply Alexander and Poincaré-Lefschetz duality to understand the homology and cohomology of M.





3. Hatcher §4.1: #2.

4. Hatcher §4.1: #4.

5. Hatcher §4.1: #11.

6. Hatcher §4.1: #15.