## Math 526: HW 6 due Monday, November 29, 2021.

1. Hatcher §4.2: #31 or #34, your choice.

2. Hatcher §4.2: #39.

3. Hatcher §4.3: #2.

4. Prove part (d) of the below. Parts (a-c) were on HW #3.

   **Poincaré duality for 3-manifolds.** Let $M$ be a closed connected orientable 3-manifold. The only interesting case of Poincaré duality here is that $H^1(M,\mathbb{Z})$ is isomorphic to $H_2(M,\mathbb{Z})$. Fill in the following outline for part of a geometric proof (all (co)homology has coeffs in $\mathbb{Z}$).

   (a) Prove that any class $x$ in $H_1(M)$ can be represented by an oriented embedded circle.

   (b) Prove that any class $y$ in $H_2(M)$ can be represented by an oriented embedded surface. That is, there is an embedded surface $S \subset M$ with $i_*([S]) = y$.

   (c) There is a bilinear pairing $H_1(M) \otimes H_2(M) \to \mathbb{Z}$, namely the intersection product (also called the homology cap product). This gives a map $H_2(M) \to H_1(M)^* = \operatorname{Hom}\big(H_1(M),\mathbb{Z}\big)$. Show that this map is injective.

   (d) Prove that $H_2(M) \to H_1(M)^* \cong H^1(M)$ is surjective, completing the proof of Poincaré duality. Hint: Use that $H^1(M;\mathbb{Z}) \cong [M,\ S^1]$.

5. Hatcher §4.3: #4.

6. Hatcher §4.3: #6.